Three Phase Theory

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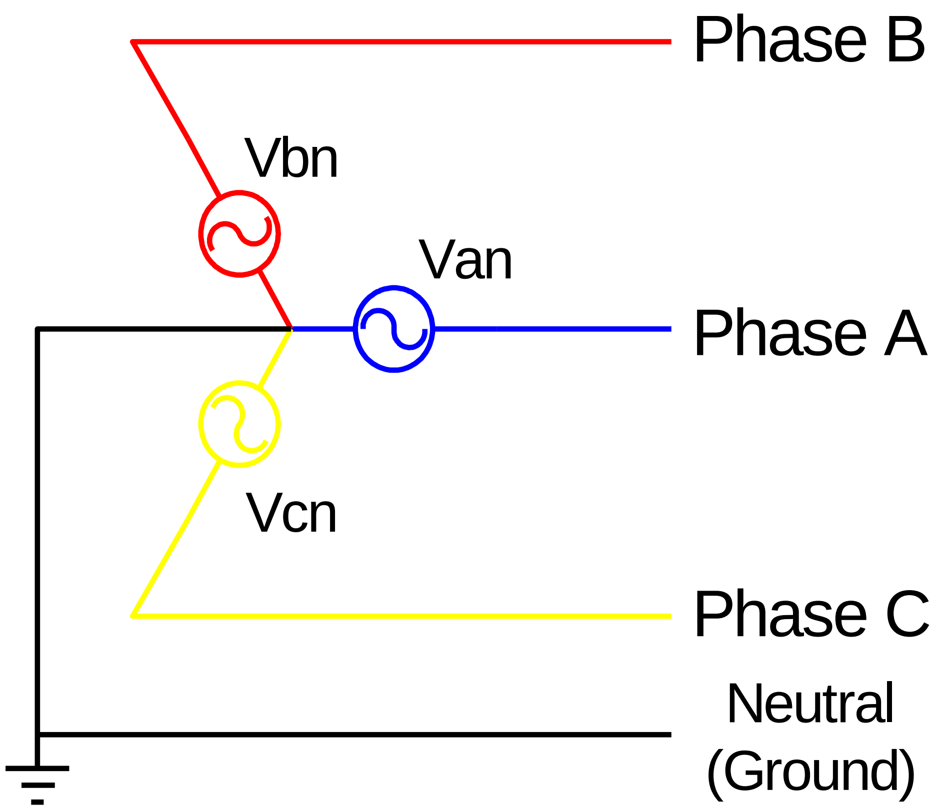
For North Carolina Electric Meter School
Tuesday, June 27, 2017 at 10:15 a.m.

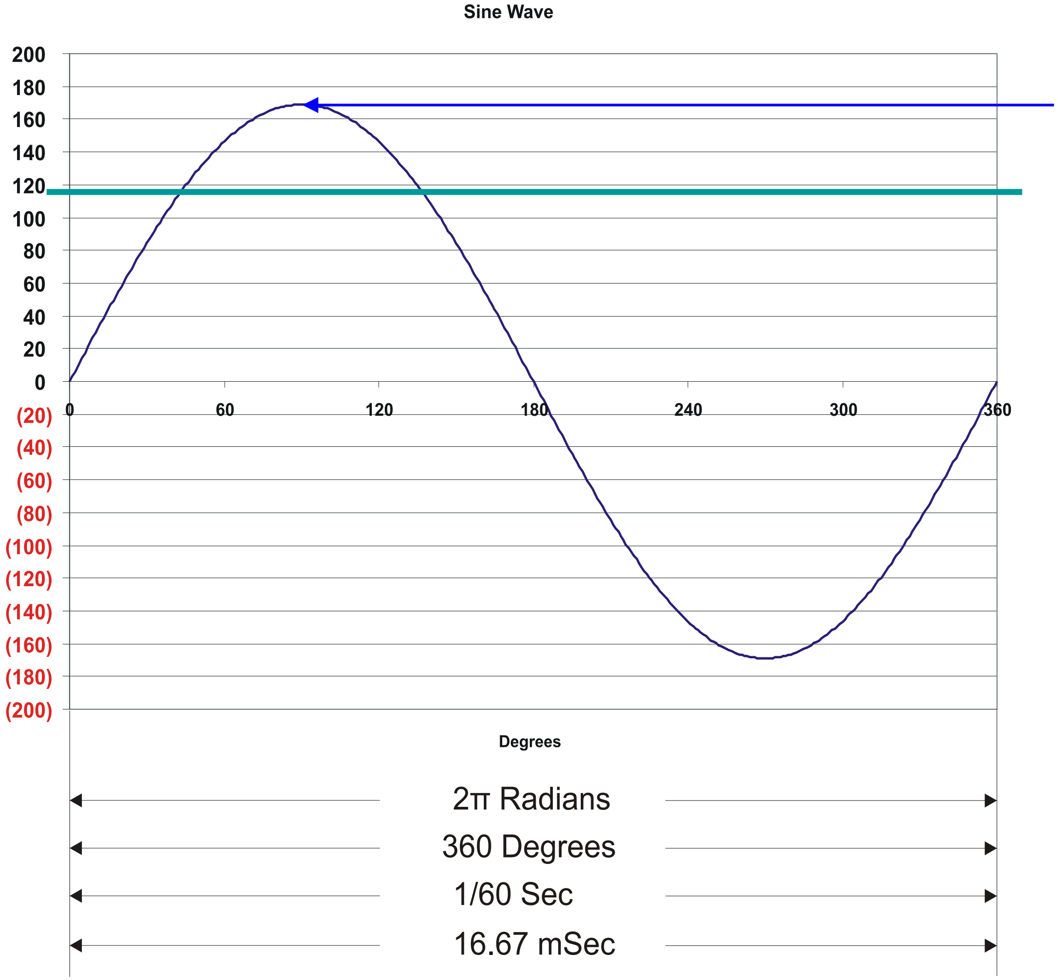
Three Phase Power
Introduction



Basic Assumptions

- Three AC voltage sources
- Voltages Displaced in time
- Each sinusoidal
- Identical in Amplitude

AC Theory – Sine Wave



$$V = V_{pk}\sin(2\pi ft - \theta)$$

$$V = \sqrt{2}V_{rms}\sin(2\pi ft - \theta)$$

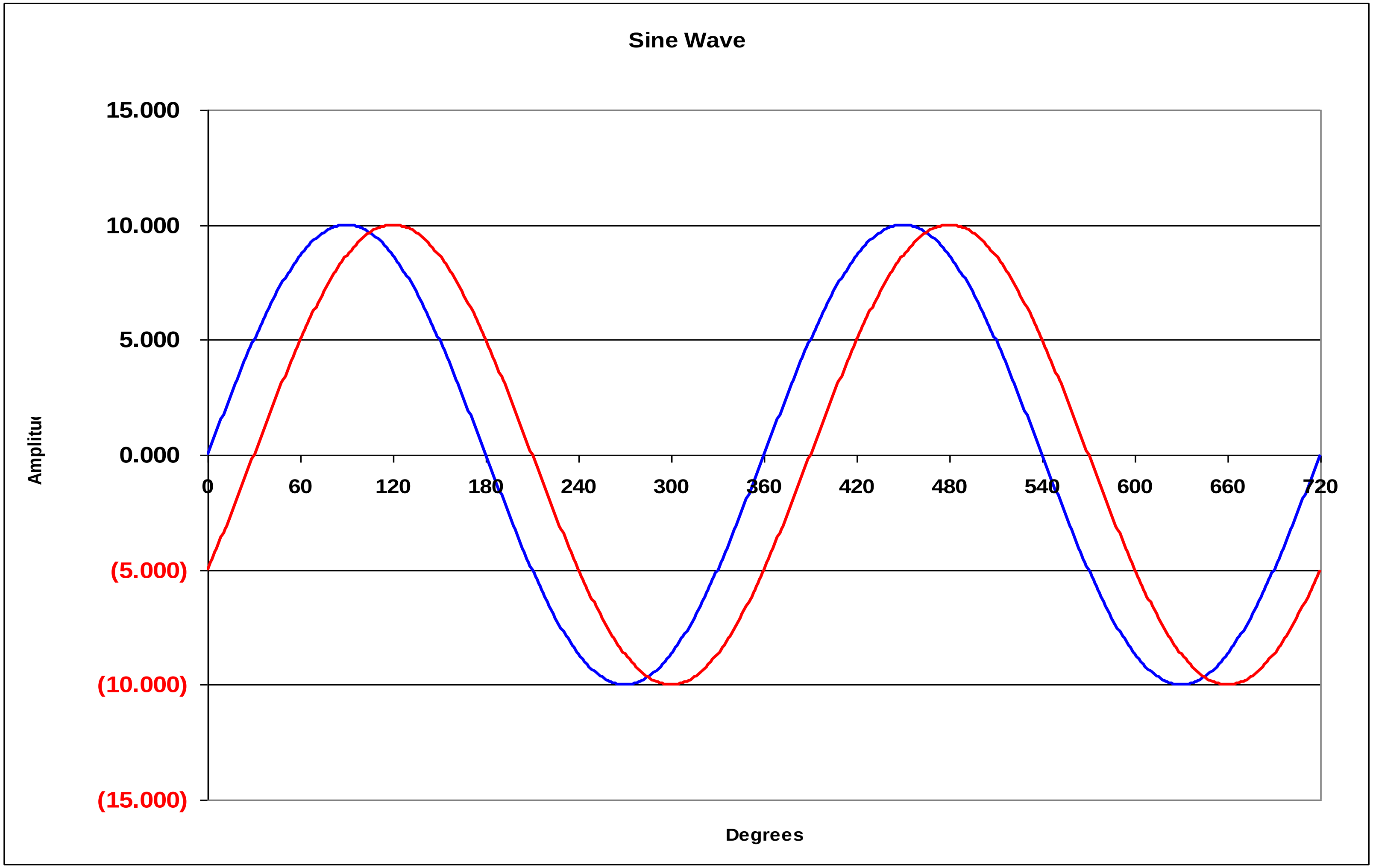
$$V_{rms} = 120$$

$$V_{pk} = 169$$

$$\theta = 0$$

$V = 10Sin(\omega t - \theta)$

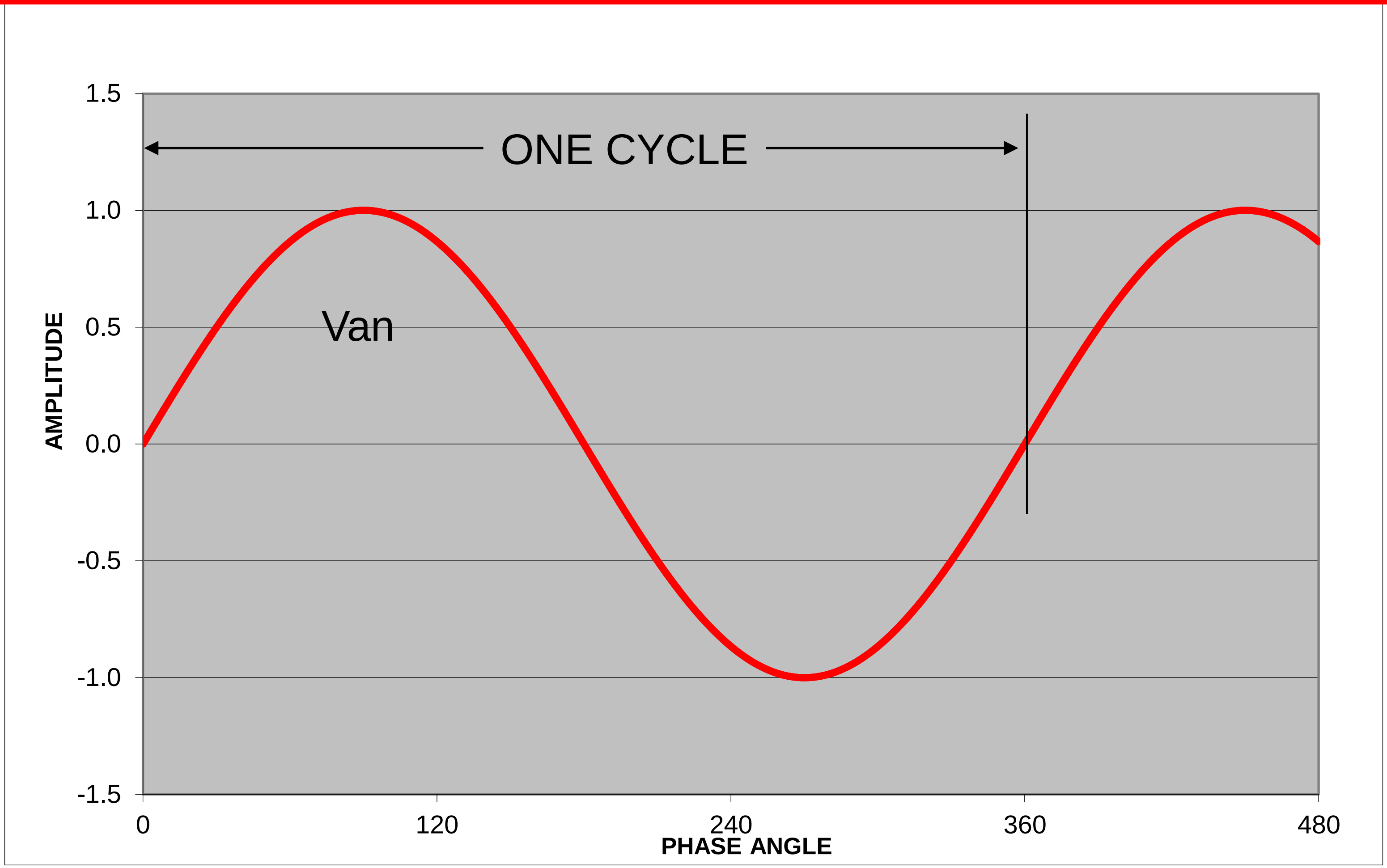
AC Theory - Phase



$\boxed{V = 10\sin(2\pi ft)}$ $\boxed{V = 10\sin(2\pi ft - 30)}$

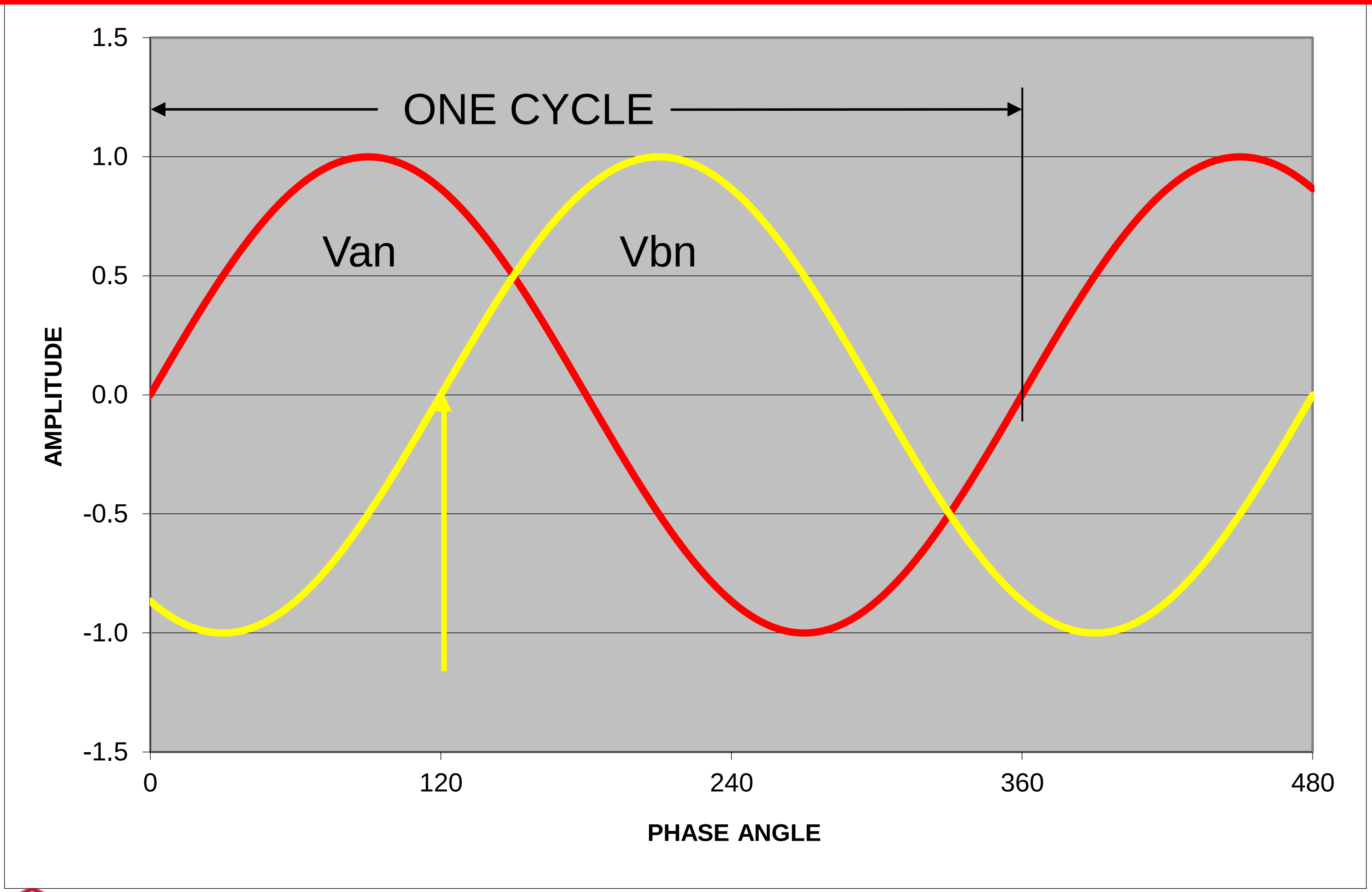
Three Phase Theory

Single Phase - Voltage Plot



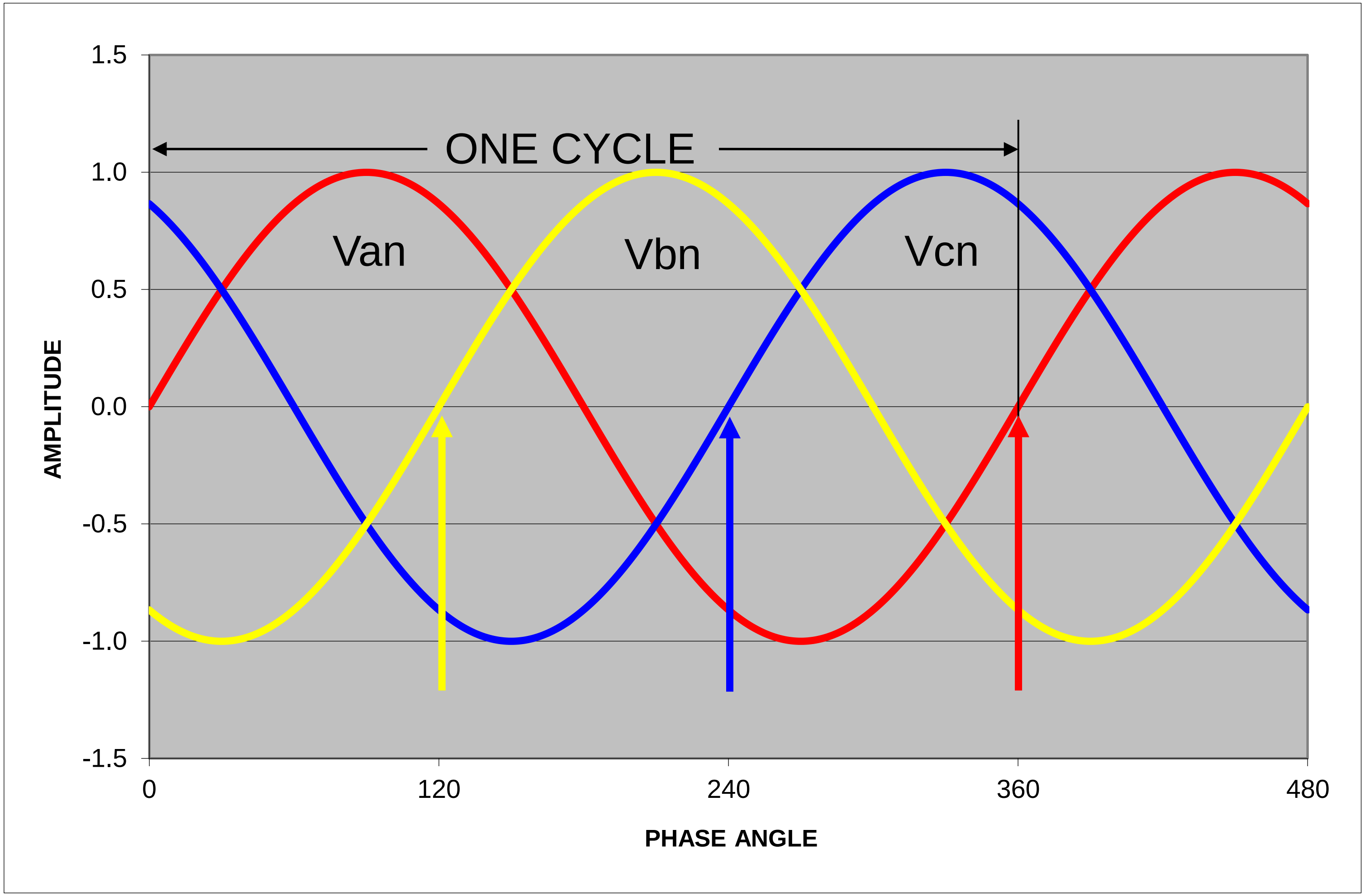
Three Phase Theory

Two Phases - Voltage Plot



Three Phase Theory

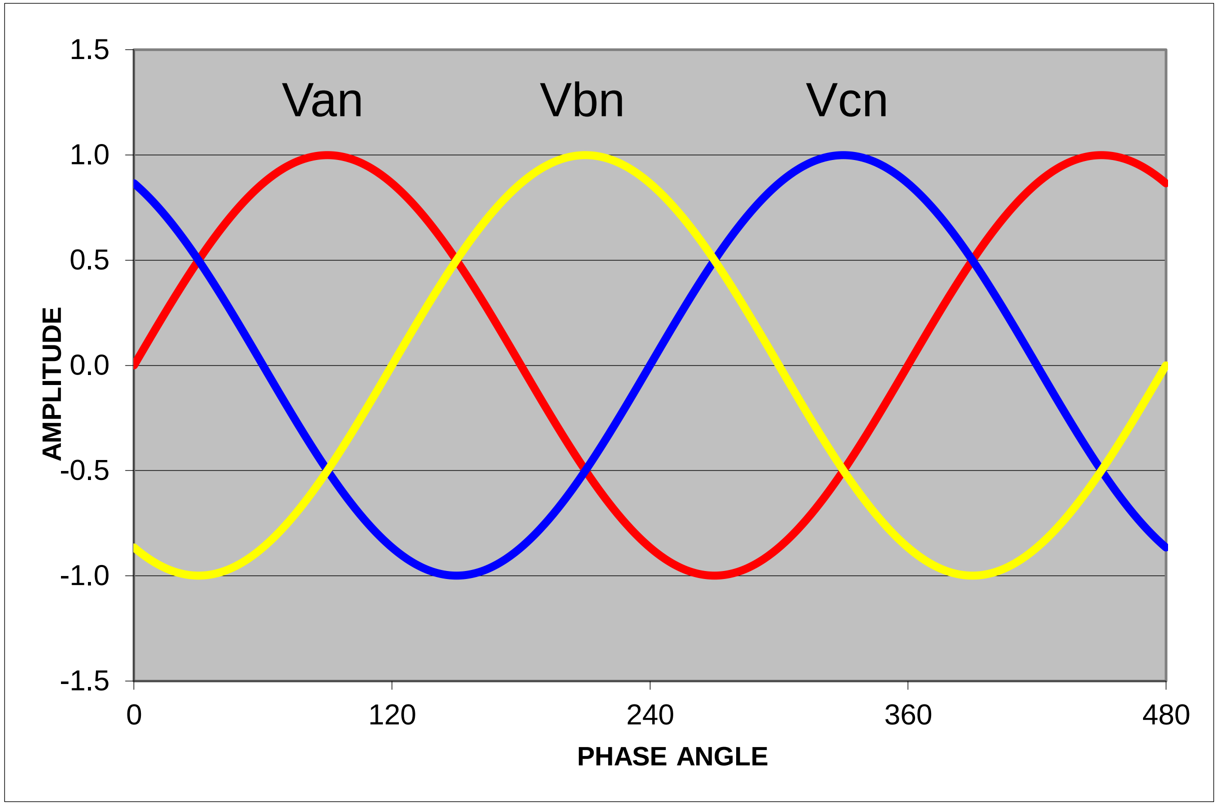
Three Phase - Voltage Plot



Three Phase Power
At the Generator

Three voltage vectors each separated by 120°.

Peak voltages essentially equal.



Most of what makes three phase systems seem complex is what we do to this simple picture in the delivery system and loads.

Three Phase Power
Basic Concept – Phase Rotation

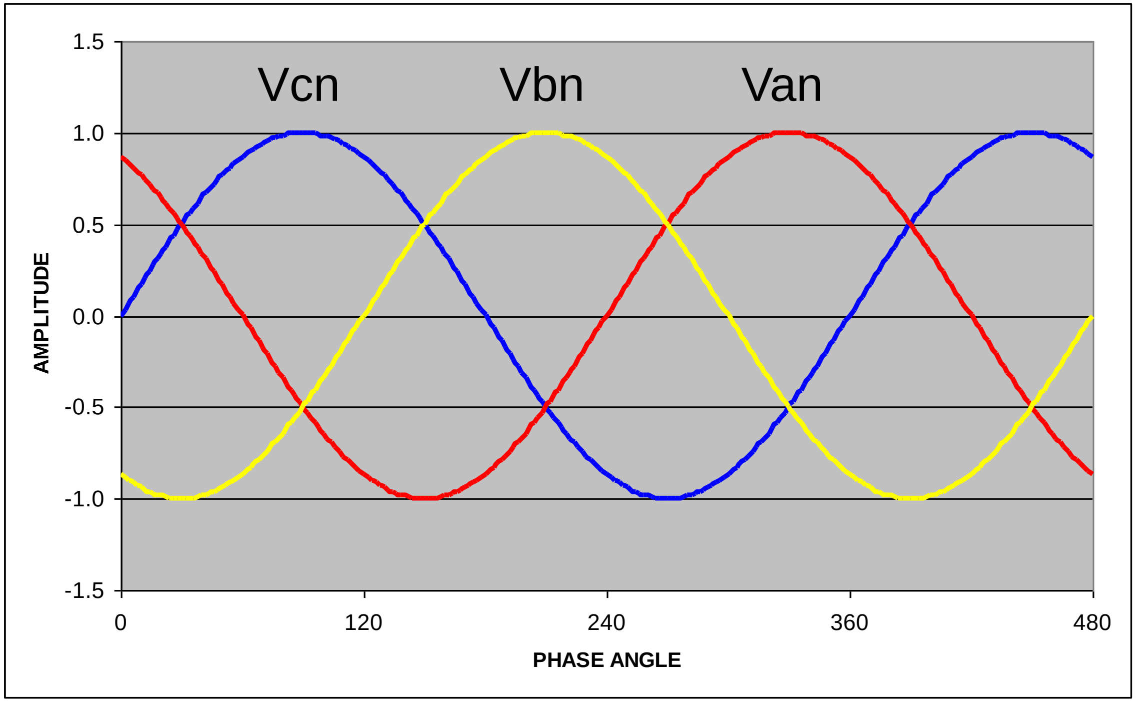
Phase Rotation:

The order in which the phases reach peak voltage.

There are only two possible sequences:

A-B-C (previous slide)

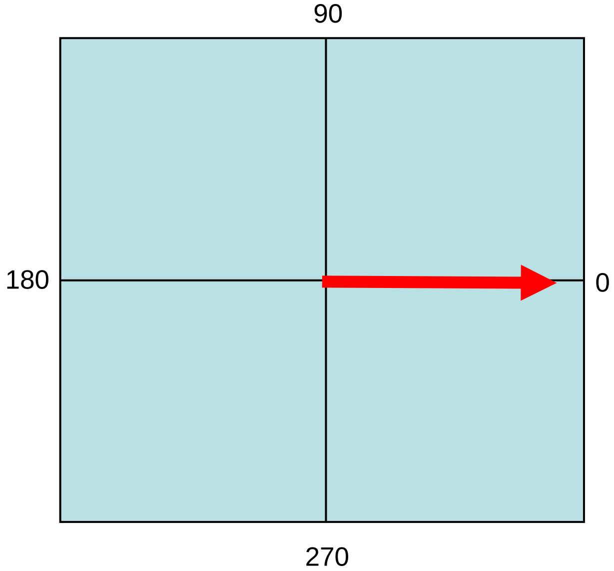
C-B-A (this slide)

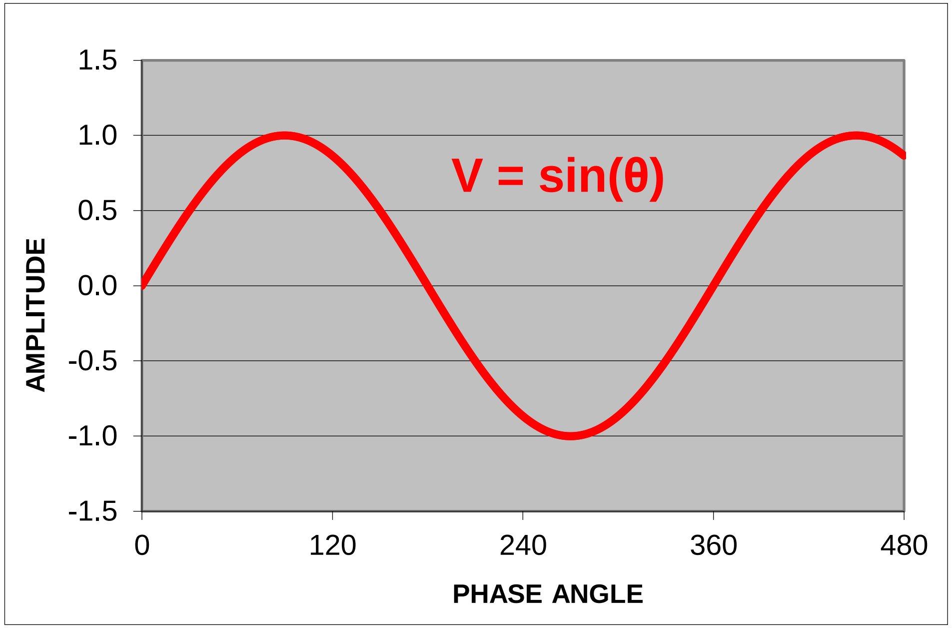


Phase rotation is important because the direction of rotation of a three phase motor is determined by the phase order.

Three Phase Theory
Phasors and Vector Notation

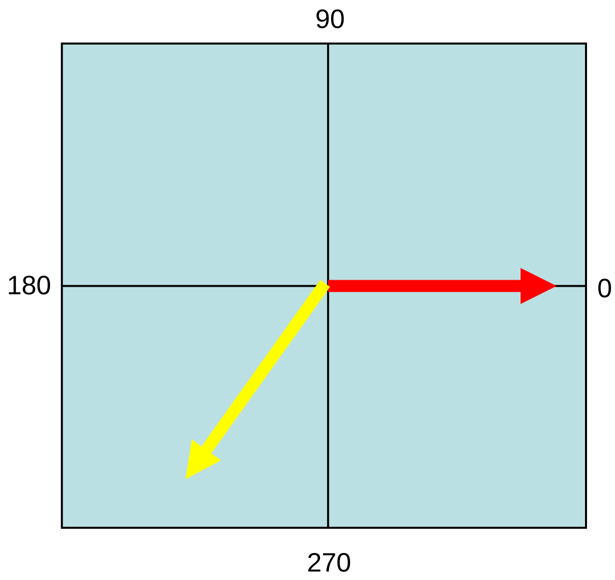
- Phasors are a graphical means of representing the amplitude and phase <u>relationships</u> of voltages and currents.

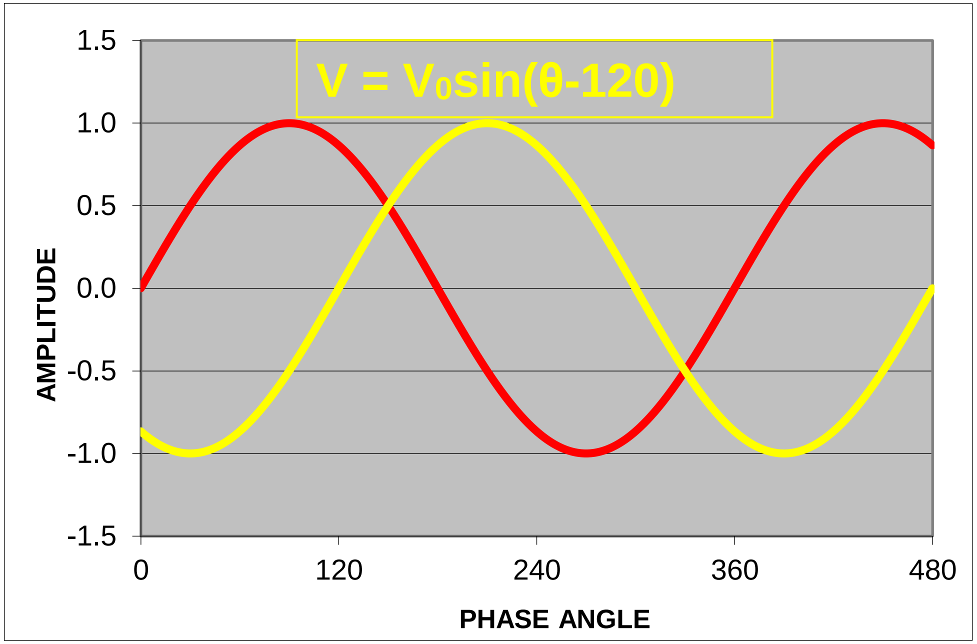




Three Phase Power
Phasors and Vector Notation

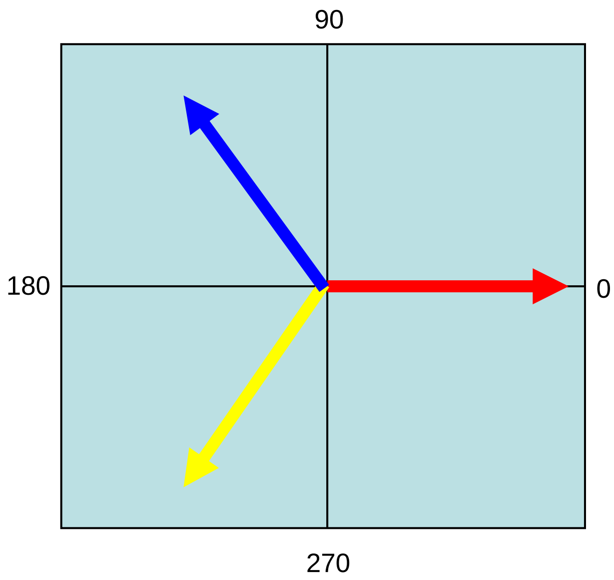
- As stated in the Handbook of Electricity Metering, by common consent, counterclockwise phase rotation has been chosen for general use in phasor diagrams.

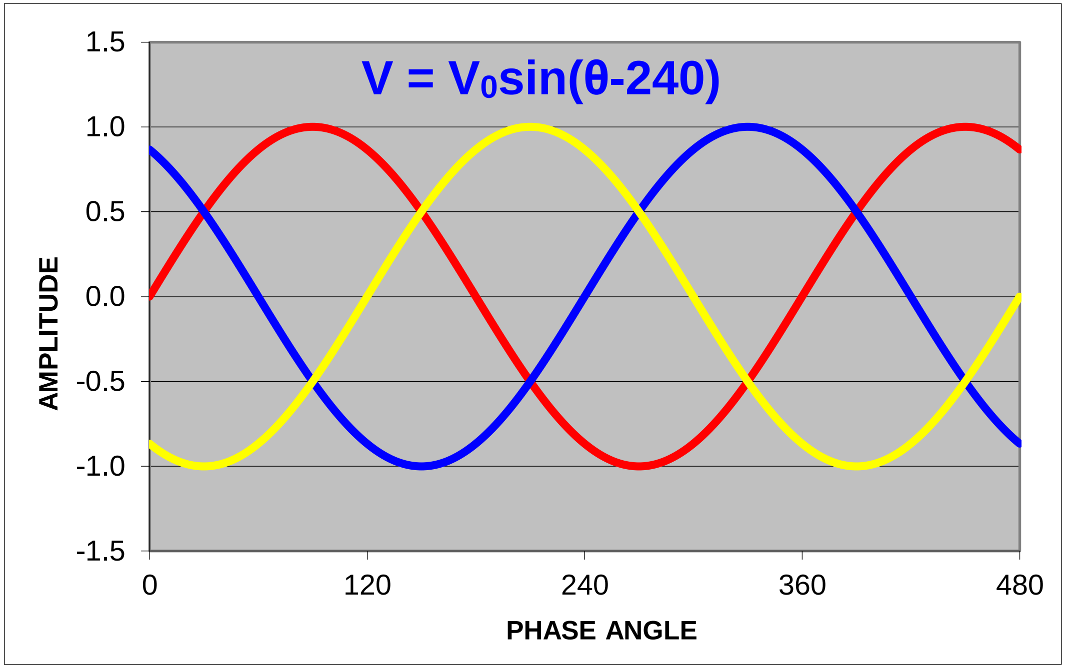




Three Phase Power
Phasors and Vector Notation

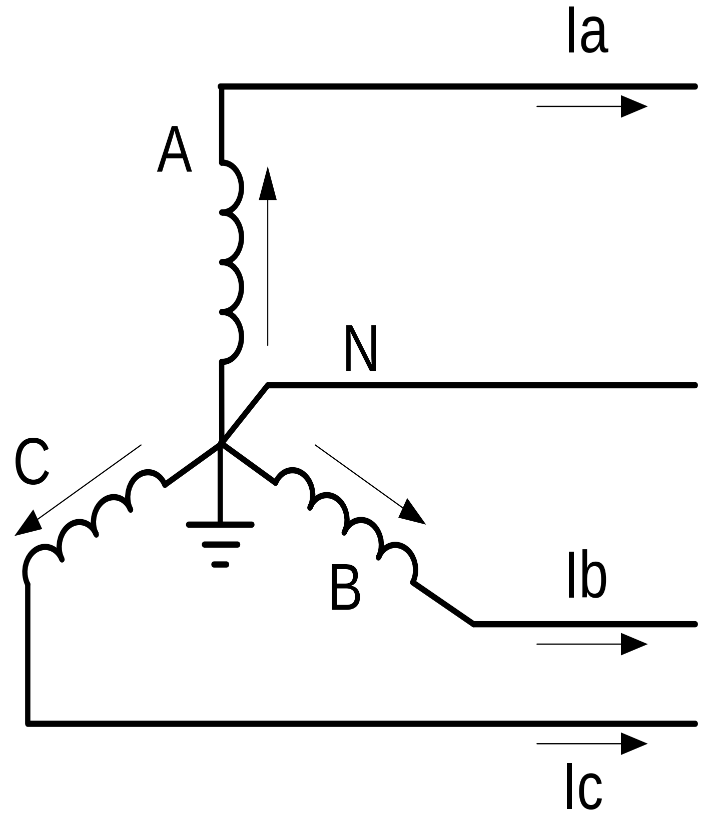
- The phasor diagram for a simple 3-phase system has three voltage phasors equally spaced at 120° intervals.
- Going clockwise the order is A – B – C.





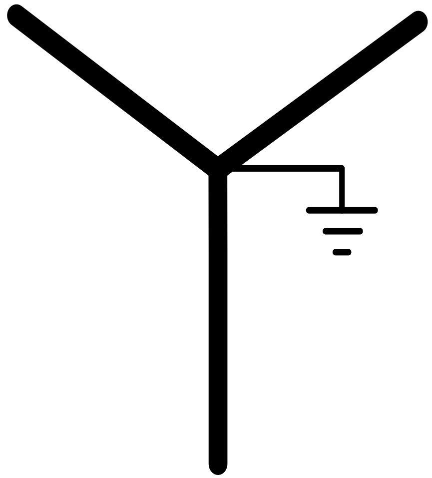
Three Phase Theory
Symbols and Conventions

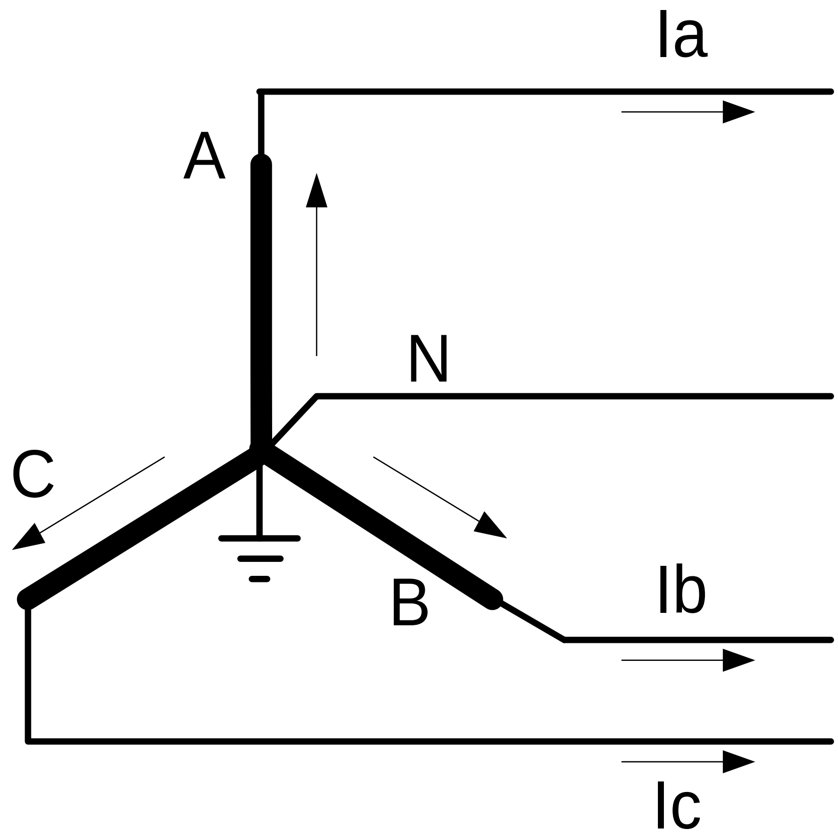
- Systems formed by interconnecting secondary of 3 single phase transformers.
- Generally primaries are not show unless details of actual transformer are being discussed.



Three Phase Theory
Symbols and Conventions

- Often even the coils are not shown but are replaced by simple line drawings

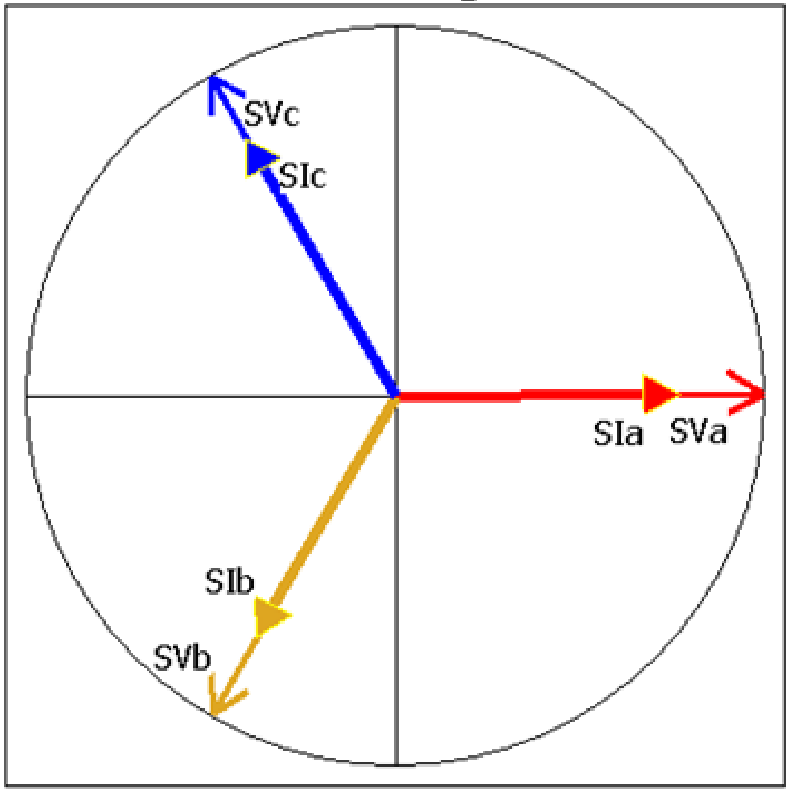




3 Phase, 4-Wire "Y" Service
0° = Unity Power Factor

- Three Voltage Phasors
- 120° Apart
- Three Current Phasors
- Aligned with Voltage at PF=1

Vector Diagram



SVa	120.707	0.00º
SIa	1.012	359.97º
PF =	1.000	-0.03º
Lead		

SVb	119.419	119.82º
SIb	0.994	119.70º
PF =	1.000	-0.12º
Lead		

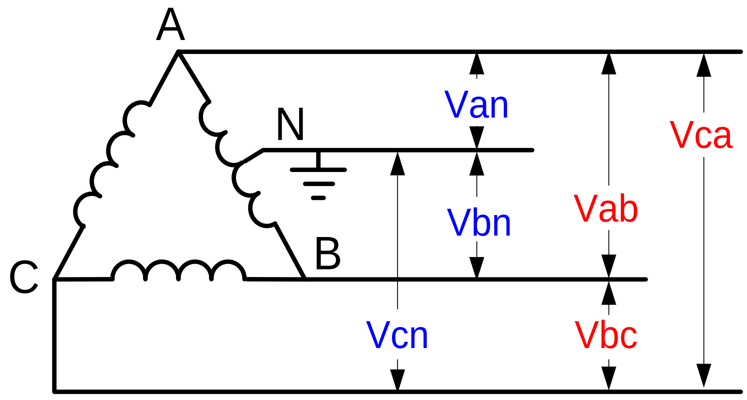
SVc	119.727	239.94º
SIc	1.056	239.96º
PF =	1.000	0.02º
Lag		

Vsys =	119.951
Isys =	1.021
PF =	1.000
ROT =	ABC

Symbols and Conventions
Labeling

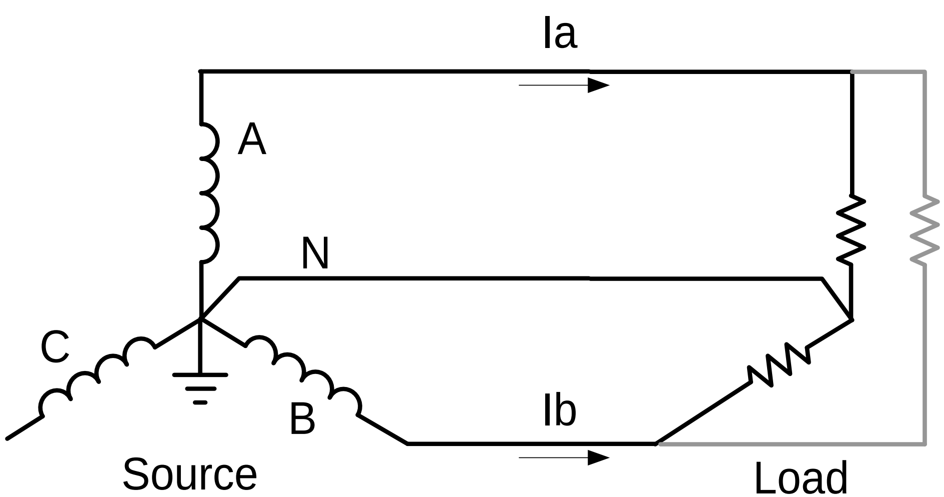
- Voltages are generally labeled Va, Vb, Vc, Vn for the three phases and neutral
- This can be confusing in complex cases.
- The recommended approach is to use two subscripts so the two points between which the voltage is measured are unambiguous.

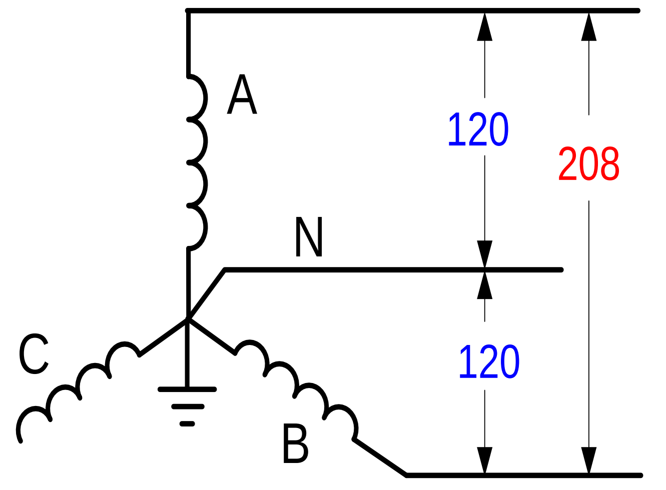
Vab means voltage at “a” as measured relative to “b”.



2 Phase, 3-Wire "Y" Service
"Network Connection"

Single phase variant of the service.





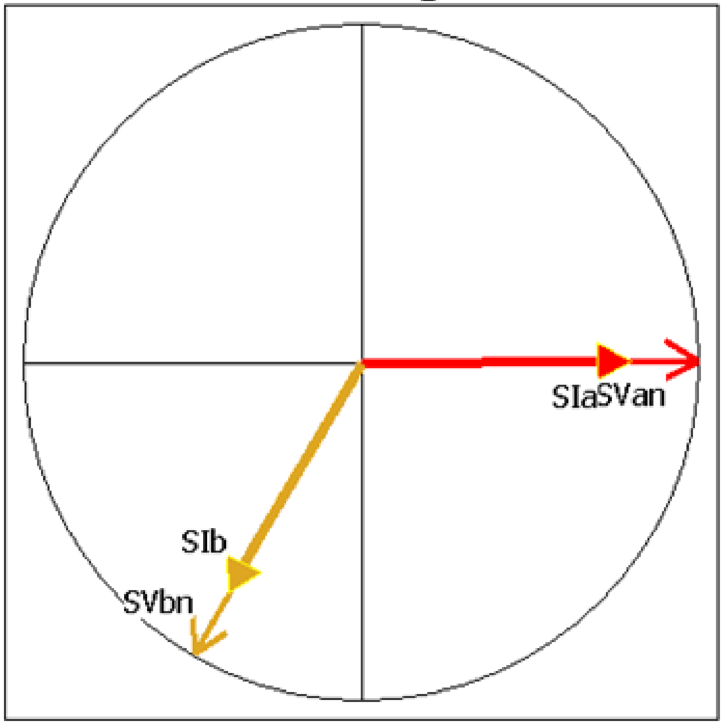
Two voltage sources with their returns connected to a common point.

Provides 208 rather than 240 volts across "high side" wires.

2 Phase, 3-Wire "Network" Service

- Two Voltage Phasors
- 120° Apart
- Two Current Phasors
- Aligned with Voltage at PF=1

Vector Diagram



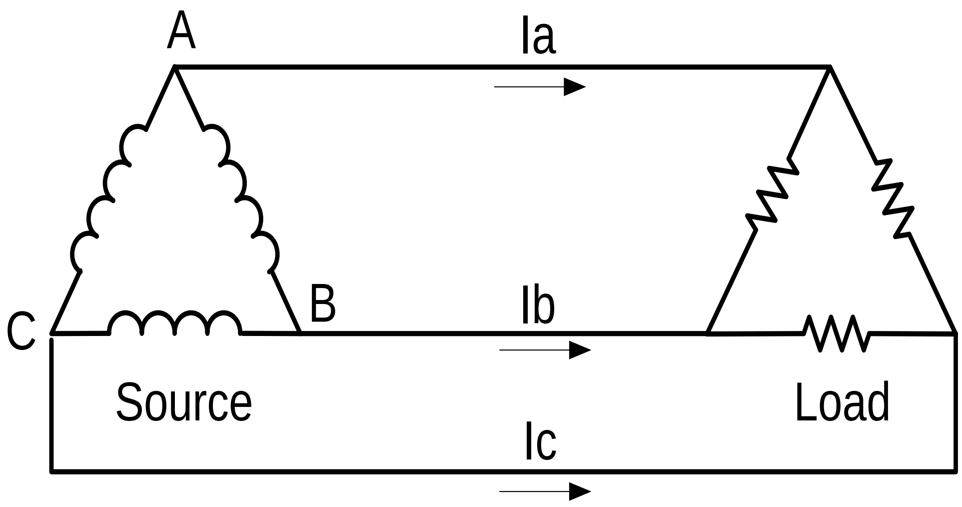
SVan	120.710	0.00°
SIa	1.012	359.99°
PF =	1.000	-0.01°
Lead		

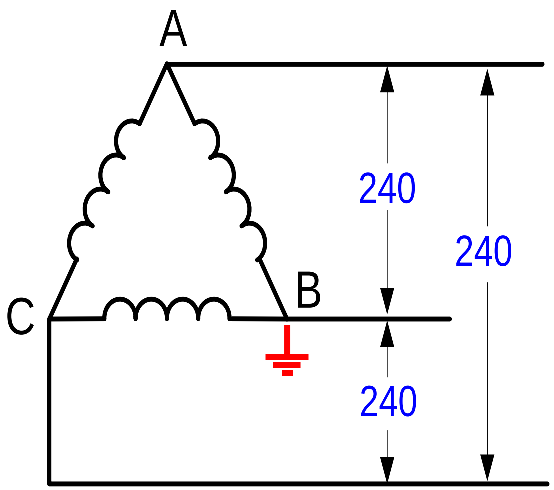
SVbn	119.411	119.82°
SIb	0.993	119.72°
PF =	1.000	-0.09°
Lead		

Vsys =	120.060
Isys =	1.003
PF =	1.000

3 Phase, 3-Wire Delta Service

Common service type for industrial customers. This service may have NO neutral.

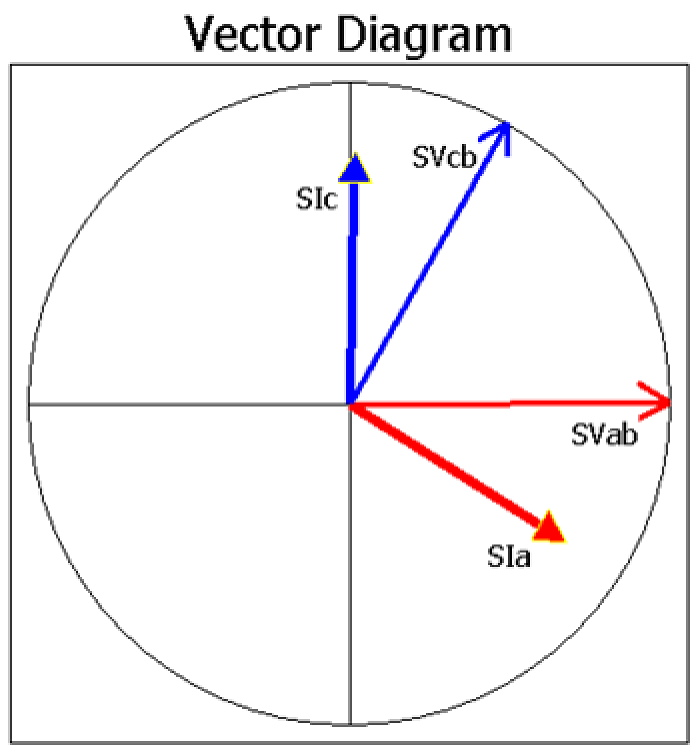




- Voltages normally measured relative to phase B.
    - Sometimes phase B will be grounded
- Voltage and current vectors do not align.
- Service is provided even when a phase is grounded.

3 Phase, 3-Wire Delta Service
Resistive Loads

- Two Voltage Phasors
- 60° Apart
- Two Current Phasors
- For a resistive load one current leads by 30° while the other lags by 30°

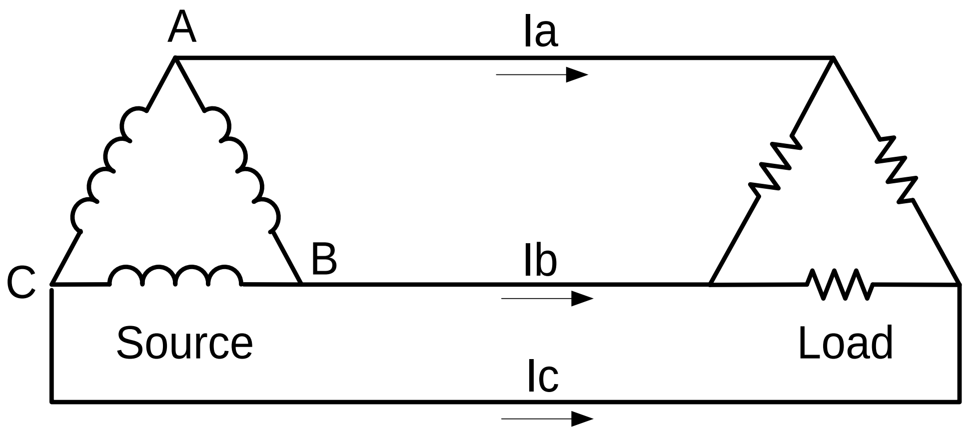


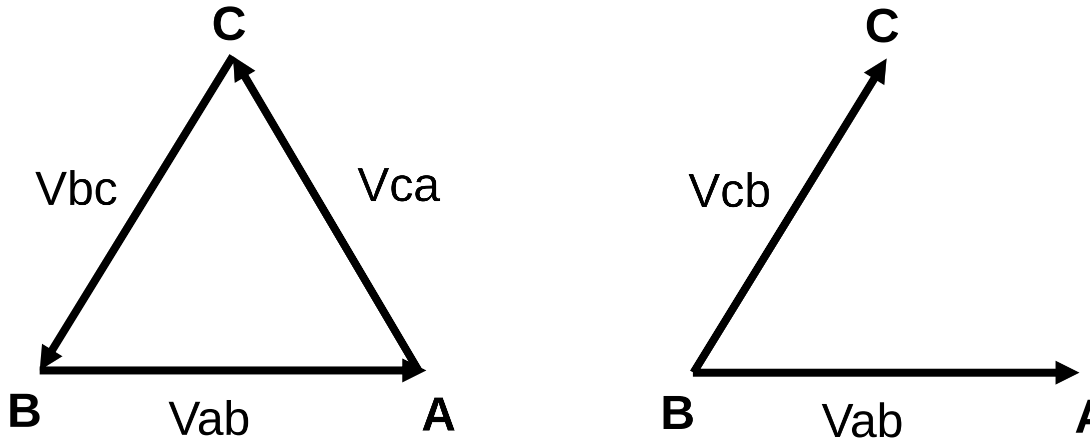
SVab	238.922	0.00º
SIa	1.055	32.74º
PF =	0.839	32.74º
Lag		

SVcb	237.914	299.48º
SIc	1.033	271.29º
PF =	0.881	-28.19º
Lead		

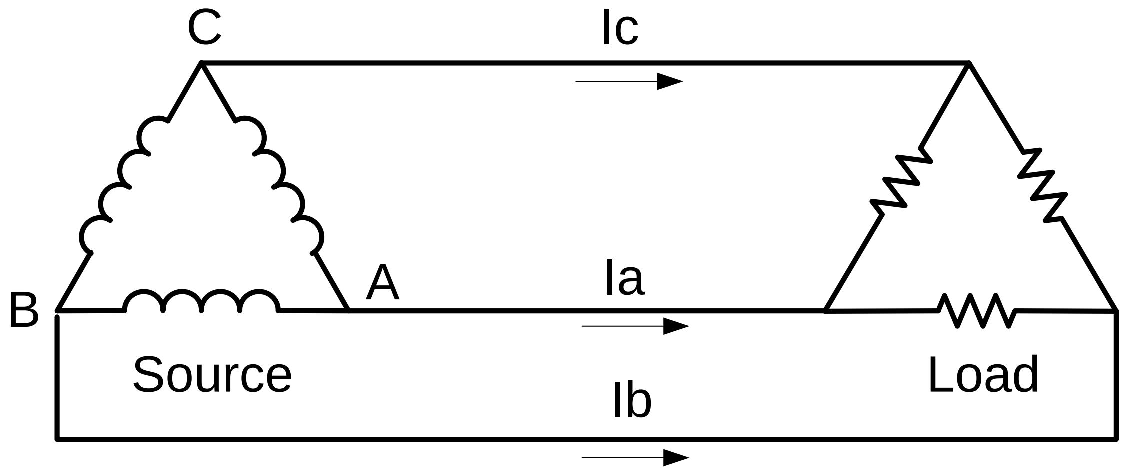
Vsys =	238.418
Isys =	1.044
PF =	0.860

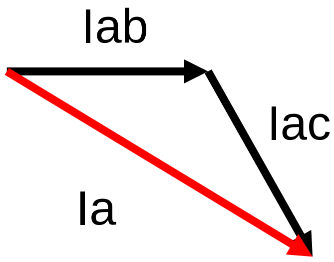
3 Phase, 3-Wire Delta Service
Understanding the Diagram

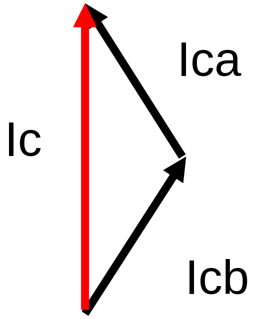




3 Phase, 3-Wire Delta Service
Understanding the Diagram



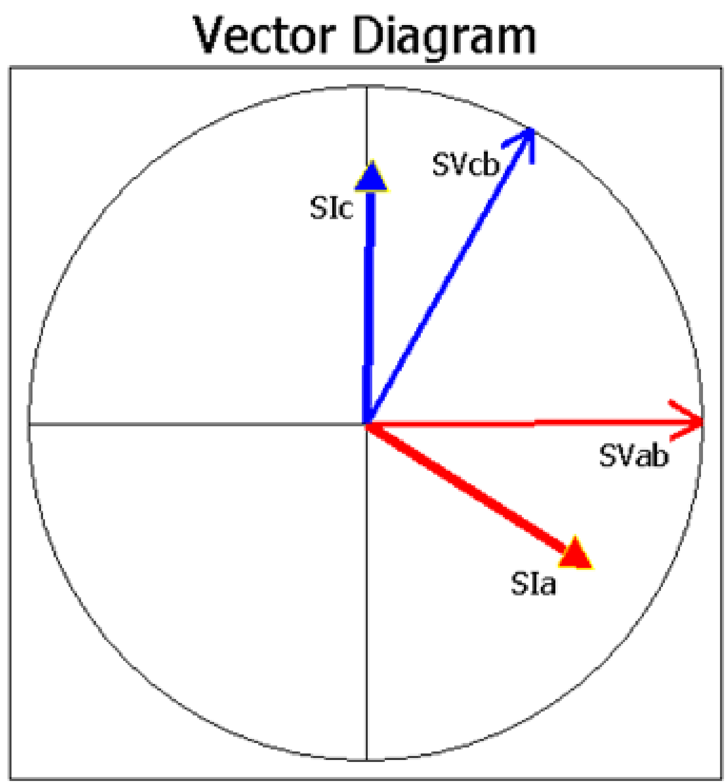




3 Phase, 3-Wire Delta Service

Resistive Load

- Two Voltage Phasors
- 60° Apart
- Two Current Phasors
- For a resistive load one current leads by 30° while the other lags by 30°



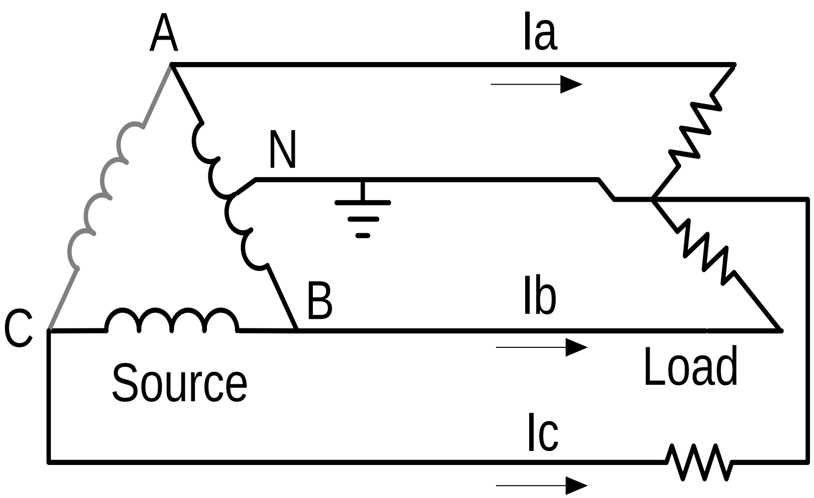
SVab	238.922	0.00º
SIa	1.055	32.74º
PF =	0.839	32.74º
Lag		

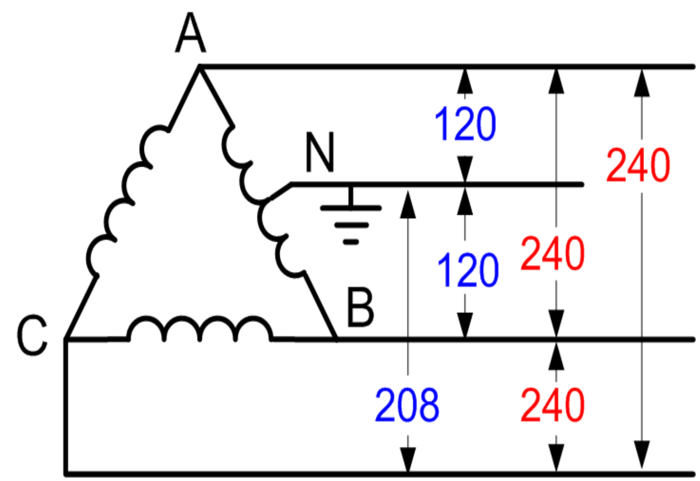
SVcb	237.914	299.48º
SIc	1.033	271.29º
PF =	0.881	-28.19º
Lead		

Vsys =	238.418
Isys =	1.044
PF =	0.860

3 Phase, 4-Wire Delta Service

Common service type for industrial customers. Provides a residential like 120/240 service (lighting service) single phase 208 (high side) and even 3 phase 240 V.



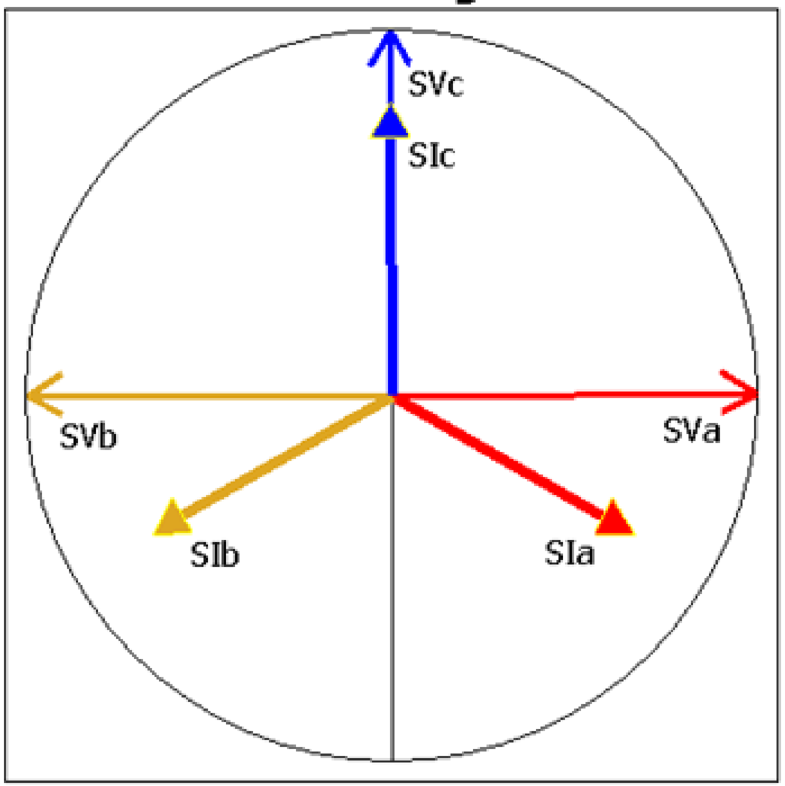


- Voltage phasors form a “T” 90° apart
- Currents are at 120° spacing
- In 120/120/208 form only the “hot” (208) leg has its voltage and current vectors aligned.

3 Phase, 4-Wire Delta Service
Resistive Load

- Three Voltage Phasors
- 90° Apart
- Three Current Phasors
- 120° apart

Vector Diagram



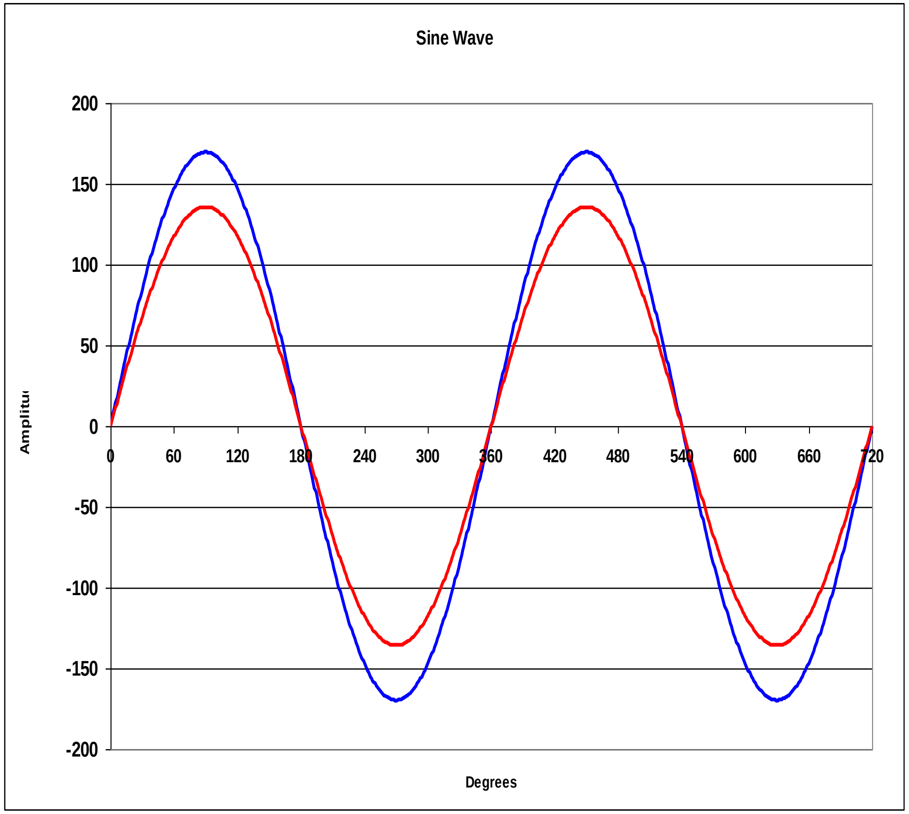
SVa	120.684	0.00º
SIa	1.013	29.97º
PF =	0.866	29.97º
Lag		

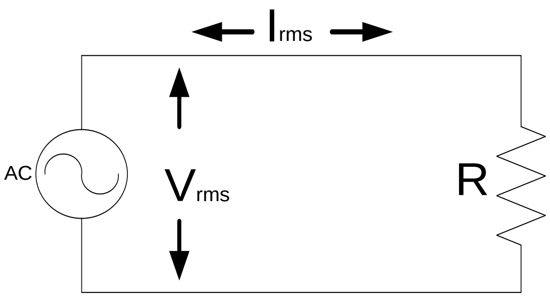
SVb	119.439	179.81º
SIb	0.994	149.68º
PF =	0.865	-30.14º
Lead		

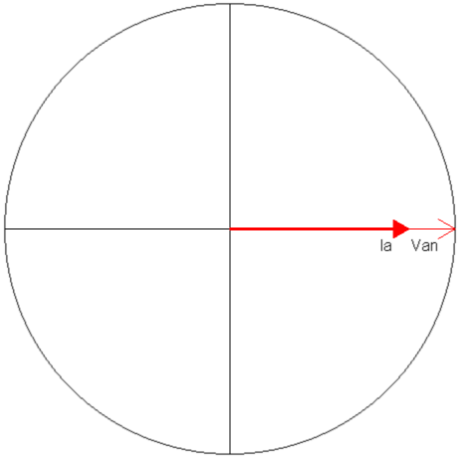
SVc	119.720	269.91º
SIc	1.056	269.97º
PF =	1.000	0.05º
Lag		

Vsys =	119.948
Isys =	1.021
PF =	0.910
ROT = ABC	

AC Theory – Resistive Load

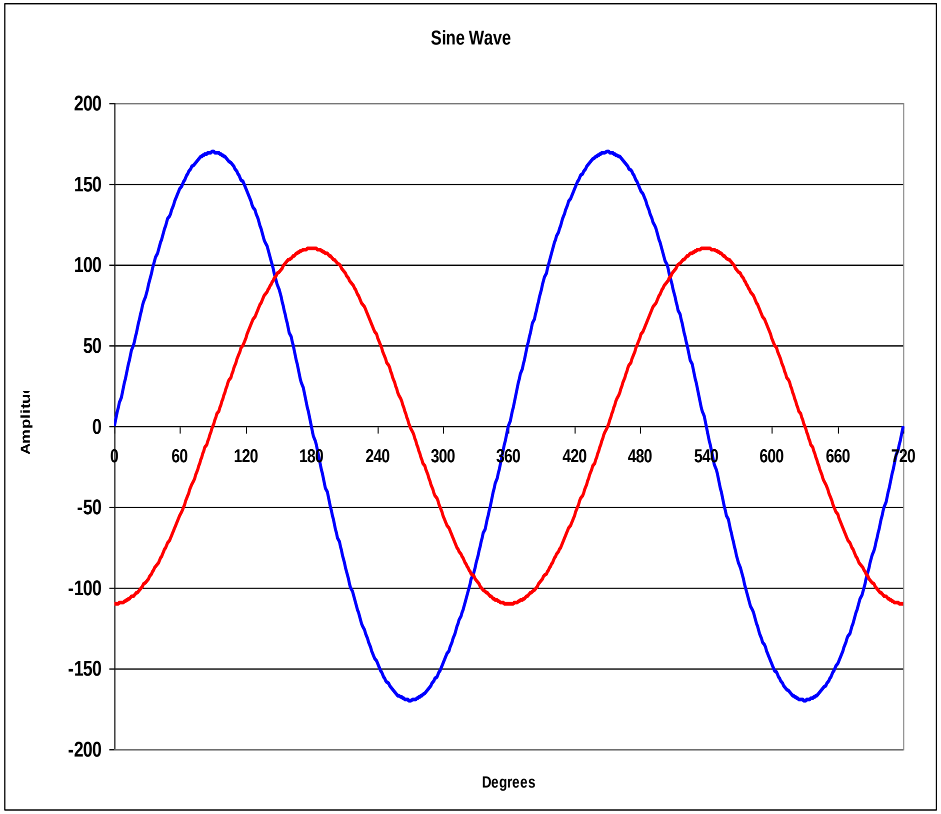


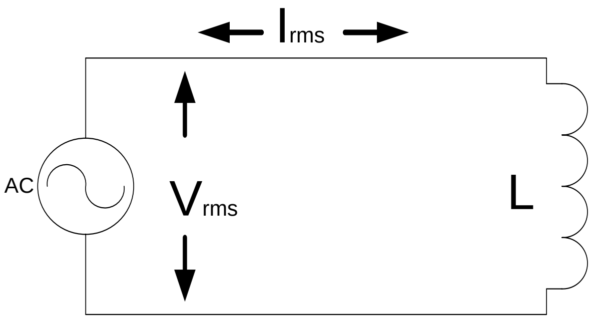


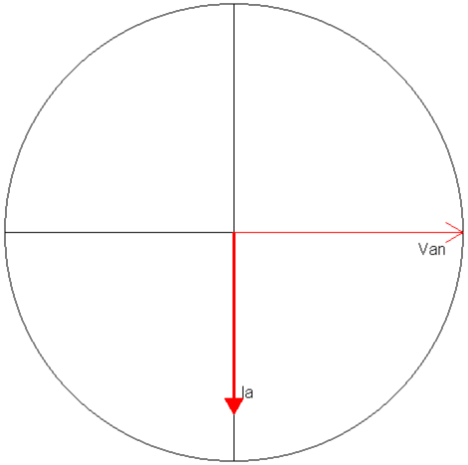


Resistors are measured in Ohms. When an AC voltage is applied to a resistor, the current is in phase. A resistive load is considered a “linear” load because when the voltage is sinusoidal the current is also sinusoidal.

AC Theory – Inductive Load

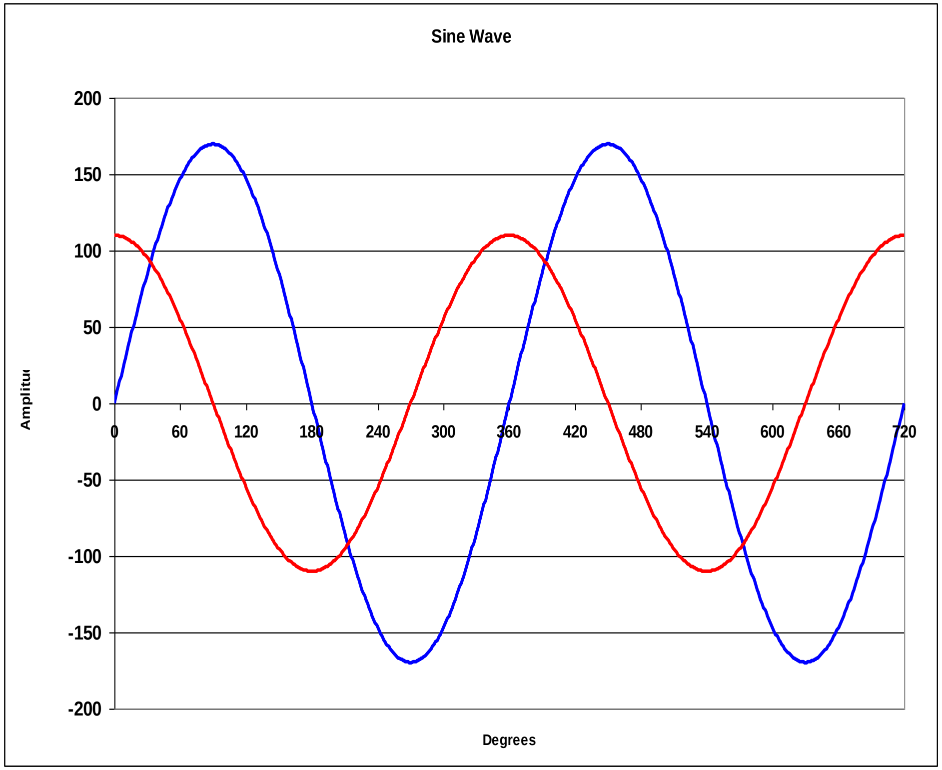


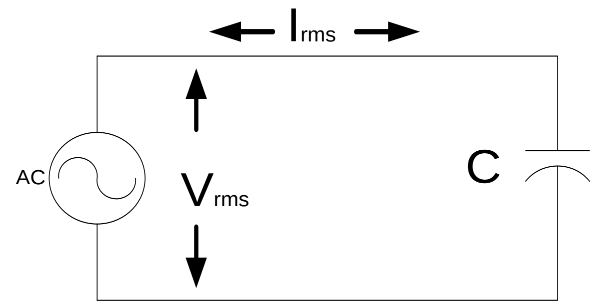


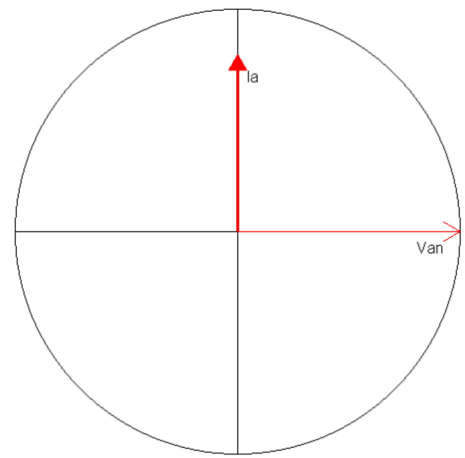


Inductors are measured in Henries. When an AC voltage is applied to an inductor, the current is 90 degrees out of phase. We say the current "lags" the voltage. A inductive load is considered a "linear" load because when the voltage is sinusoidal the current is also sinusoidal.

AC Theory – Capacitive Load







Capacitors are measured in Farads. When an AC voltage is applied to a capacitor, the current is 90 degrees out of phase. We say the current "leads" the voltage. A capacitive load is considered a "linear" load because when the voltage is sinusoidal the current is sinusoidal.

AC Theory – Power

- Power is defined as $P = VI$
- Since the voltage and current at every point in time for an AC signal is different, we have to distinguish between instantaneous power and average power. Generally when we say “power” we mean average power.
- Average power is only defined over an integral number of cycles.

Time Out for Trig

(Right Triangles)

The Right Triangle:

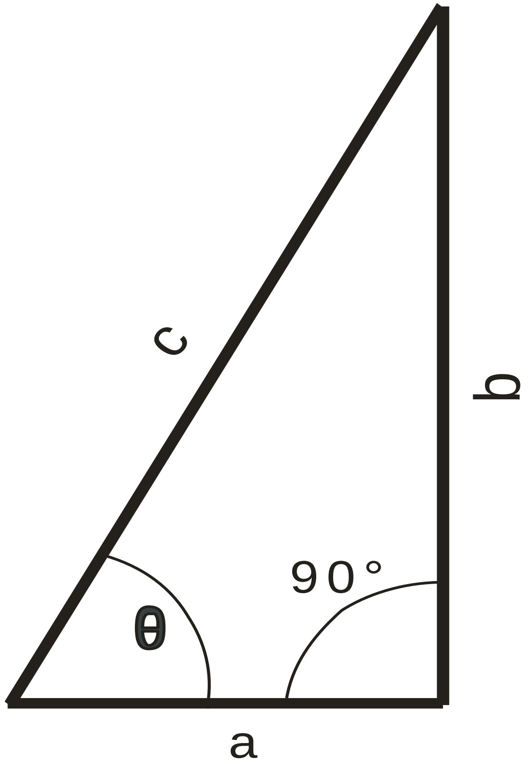
The Pythagorean theory

$$c^2 = a^2 + b^2$$

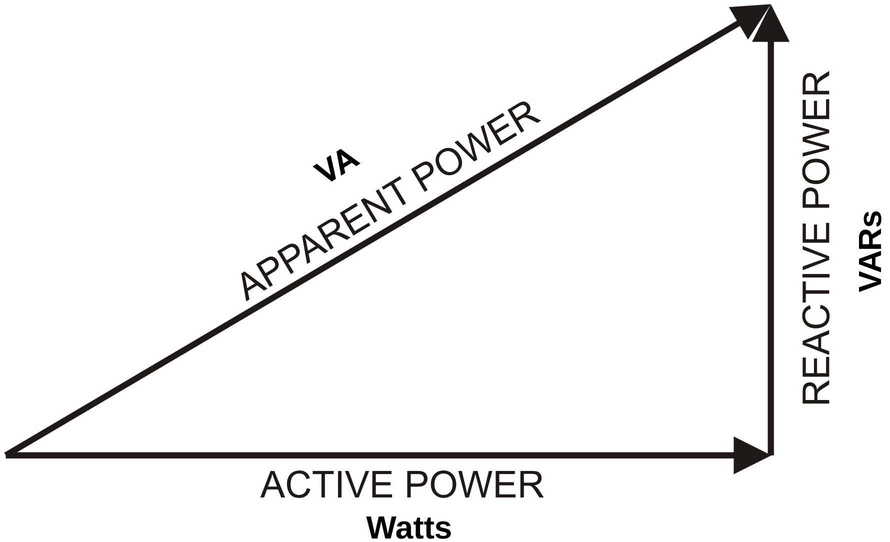
$$\sin(\theta) = \frac{b}{c}$$

$$\cos(\theta) = \frac{a}{c}$$

$$\tan(\theta) = \frac{b}{a}$$



AC Theory – Power Triangle
(Sinusoidal Waveforms)

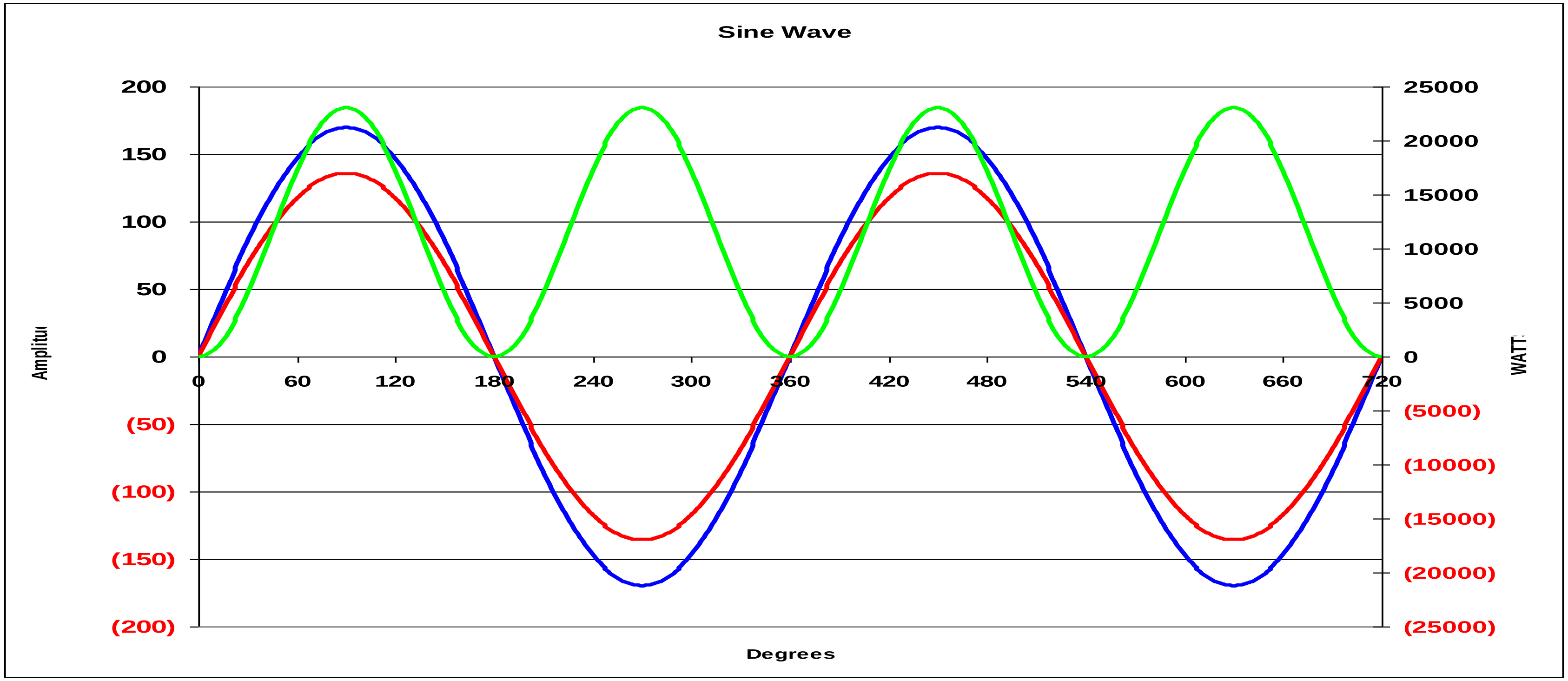


If $V = \sin(\omega t)$ and $I = \sin(\omega t - \theta)$ (the load is linear) then

Active Power =	$VI\cos(\theta)$	Watts
Reactive Power =	$VI\sin(\theta)$	VARs
Apparent Power =	VI	VA
Power Factor =	Active/Apparent = $\cos(\theta)$	

AC Theory
Instantaneous Power

For a resistive load: $p = vi = 2VI\sin^2(\omega t) = VI(1-\cos(2\omega t))$



$V = 120\sqrt{2}\sin(2\pi ft)$

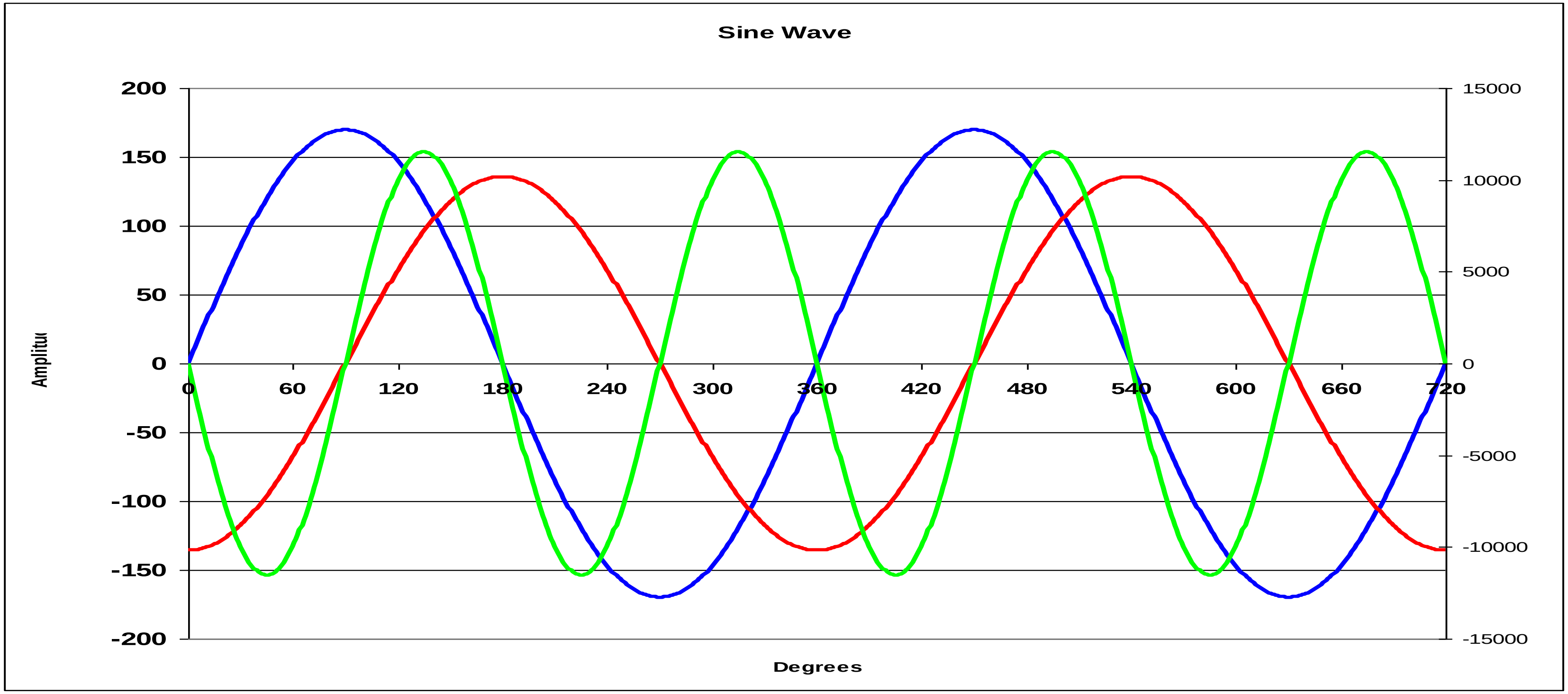
$I = 96\sqrt{2}\sin(2\pi ft)$

$P = 23040\sin^2(2\pi ft)$

P = 11520 Watts

AC Theory
Instantaneous Power

For an inductive load: $p = vi = 2VI\sin(\omega t)\sin(\omega t - 90) = -VI\sin(2\omega t)$



$V = 120\sqrt{2}\sin(2\pi ft)$

$I = 96\sqrt{2}\sin(2\pi ft - 90)$

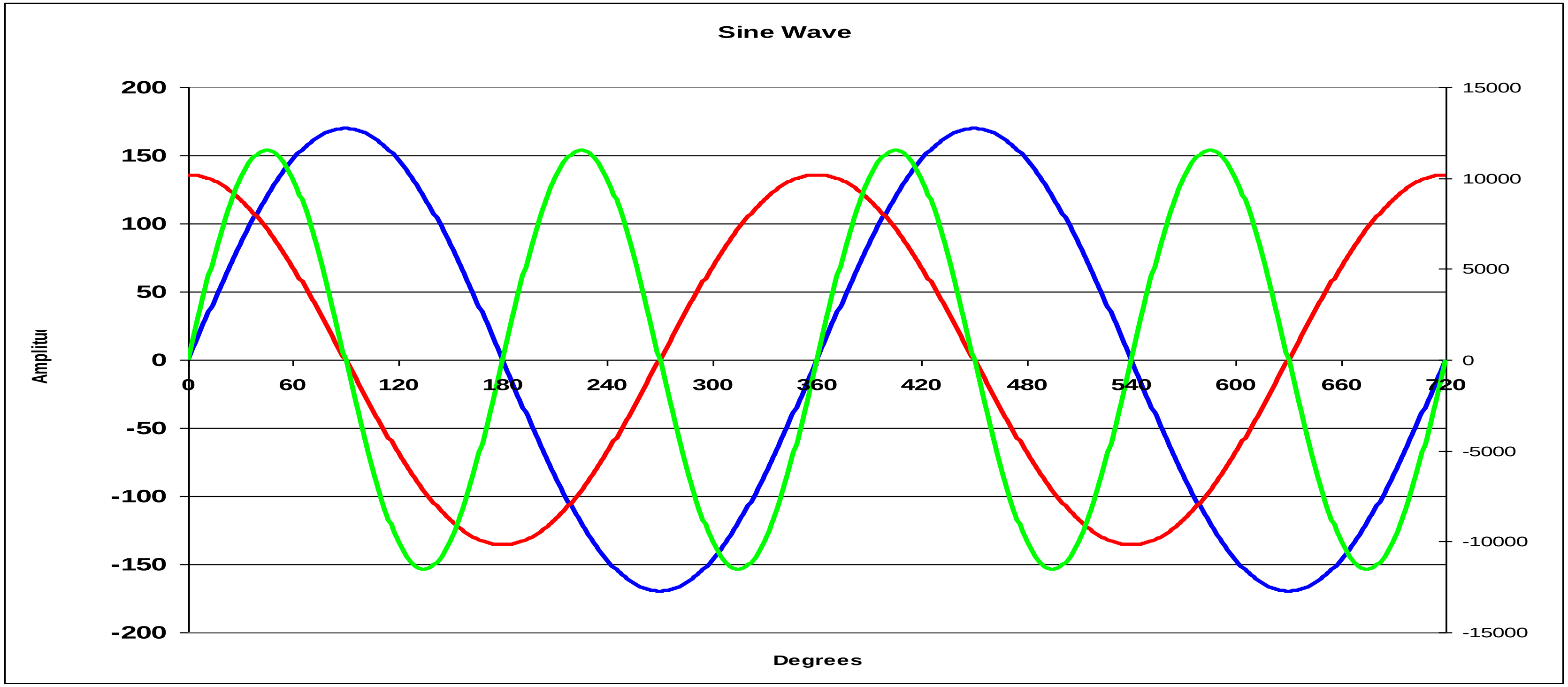
$P = -11520\sin(2\pi ft)$

P = 0 Watts

AC Theory

Instantaneous Power

For a capacitive load: $p = vi = 2VI\sin(\omega t)\sin(\omega t + 90) = VI\sin(2\omega t)$



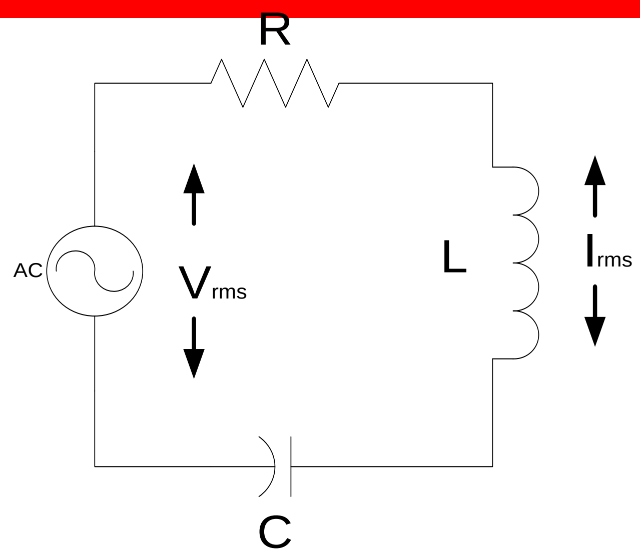
$V = 120\sqrt{2}\sin(2\pi f t)$

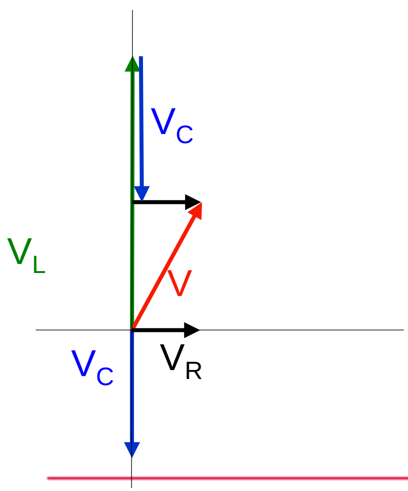
$I = 96\sqrt{2}\sin(2\pi f t + 90)$

$P = 11520\sin(2\pi f t)$

P = 0 Watts

AC Theory – Complex Circuits





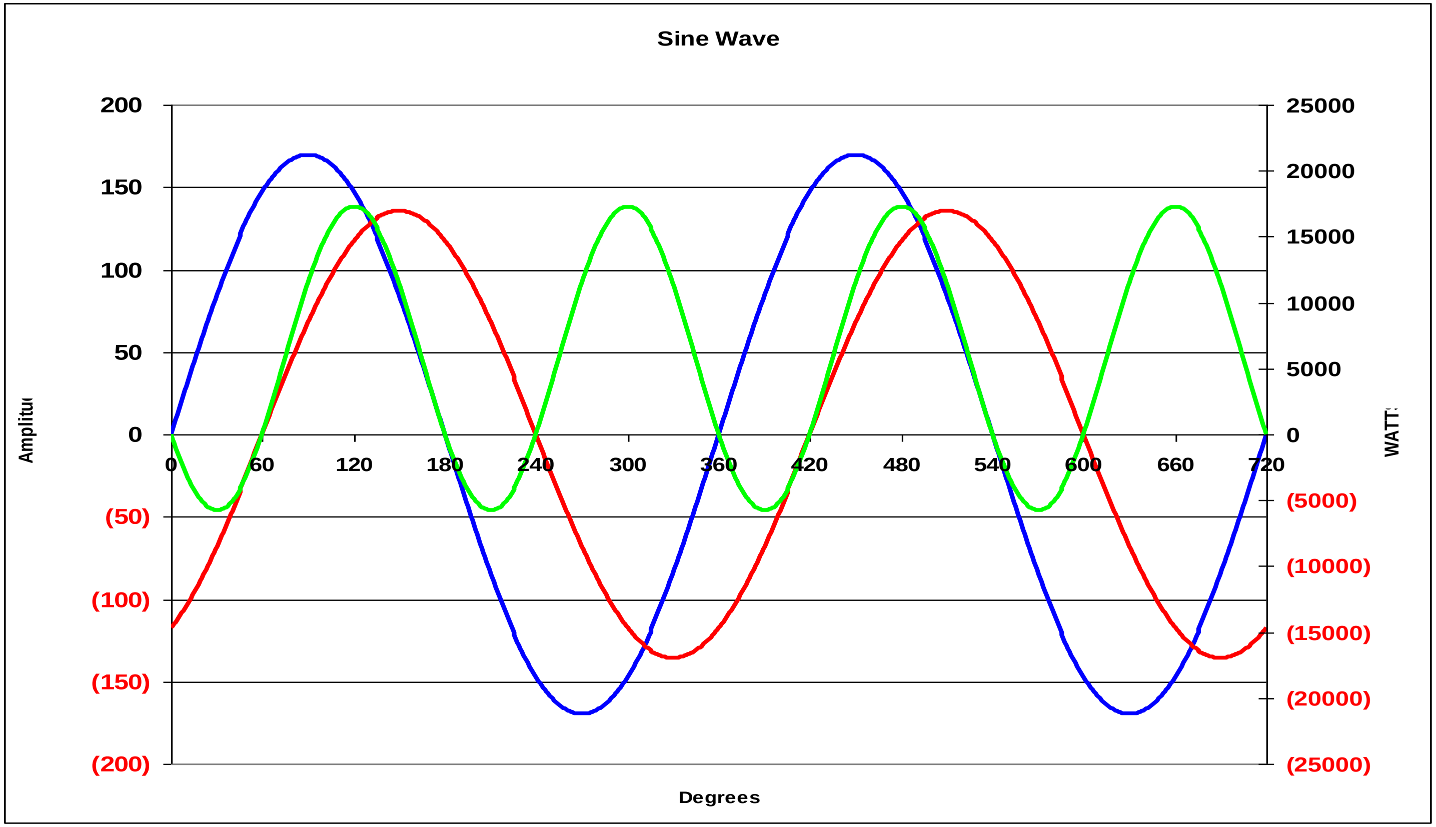
Amplitude (Current)

$$I = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

Phase (Current)

$$\varphi = \arctan\left[\frac{(\omega L - \frac{1}{\omega C})}{R}\right]$$

AC Theory – Instantaneous Power



$$V = 120\sqrt{2}\sin(2\pi f t)$$

$$I = 96\sqrt{2}\sin(2\pi f t - 60)$$

$$P = VI = 23040(\cos(60°) + \cos(4\pi f t - 60°)) = 19953 - 23040\cos(4\pi f t - 60°)$$

Three Phase Power
Blondel's Theorem

If energy be supplied to any system of conductors through N wires, the total power in the system is given by the algebraic sum of the readings of N wattmeters, so arranged that each of the N wires contains one current coil, the corresponding voltage coil being connected between that wire and some common point. If this common point is on one of the N wires, the measurement may be made by the use of N-1 wattmeters.

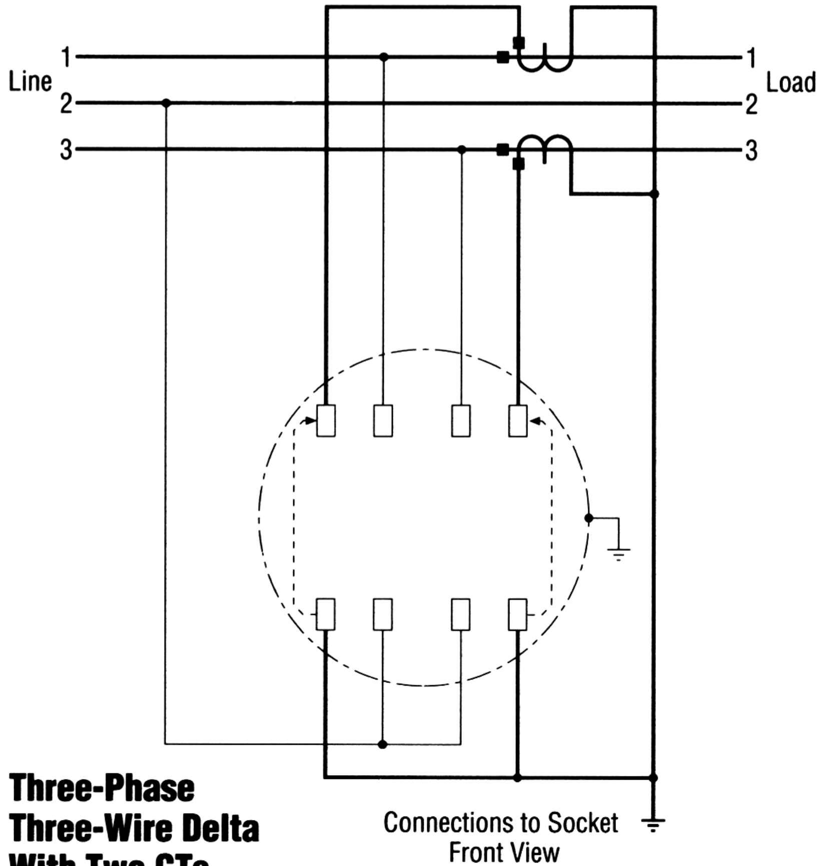
Three Phase Power
Blondel's Theorem

- Simply – We can measure the power in a N wire system by measuring the power in N-1 conductors.
- For example, in a 4-wire, 3-phase system we need to measure the power in 3 circuits.

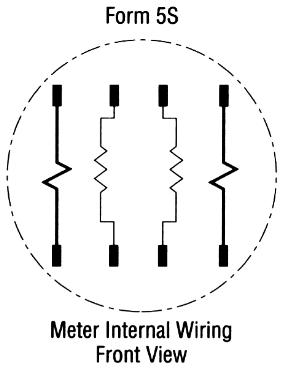
Three Phase Power
Blondel's Theorem

- If a meter installation meets Blondel's Theorem then we will get accurate power measurements under all circumstances.
- If a metering system does not meet Blondel's Theorem then we will only get accurate measurements if certain assumptions are met.

Blondel's Theorem



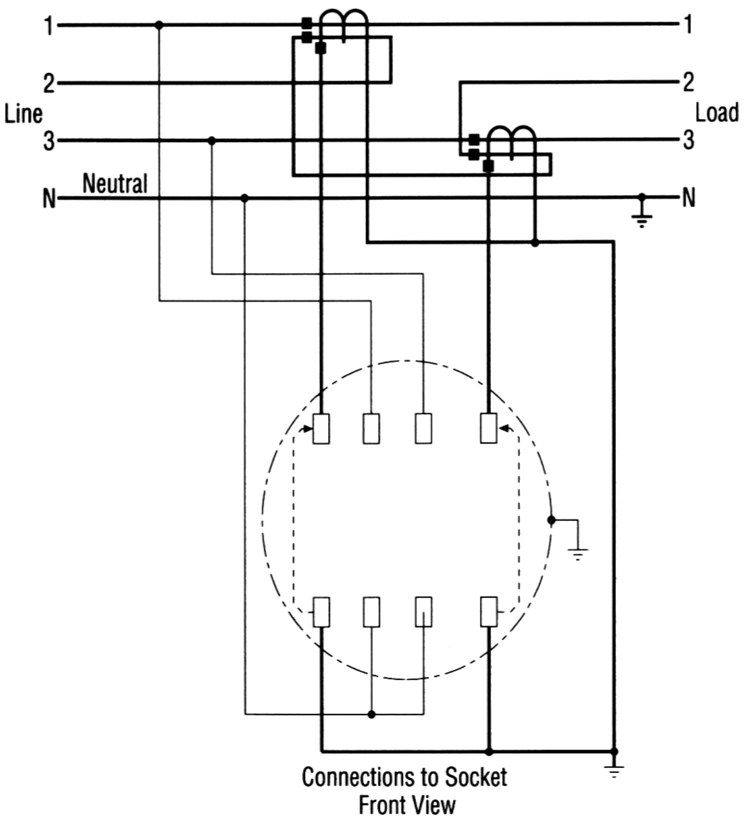
Three-Phase Three-Wire Delta With Two CTs

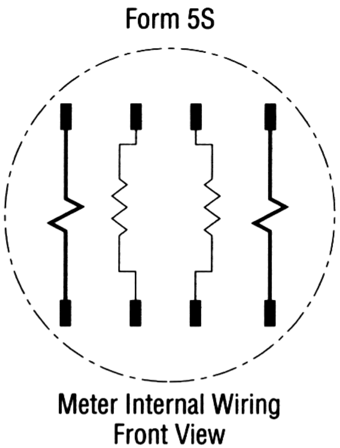


- Three wires
- Two voltage measurements with one side common to Line 2
- Current measurements on lines 1 & 3.

This satisfies Blondel's Theorem.

Blondel's Theorem





Three-Phase
Four-Wire Wye
With Two Equal-Ratio CTs

- Four wires
- Two voltage measurements to neutral
- Current measurements on lines 1 & 3. How about line 2?

This DOES NOT satisfy Blondel's Theorem.

Blondel’s Theorem

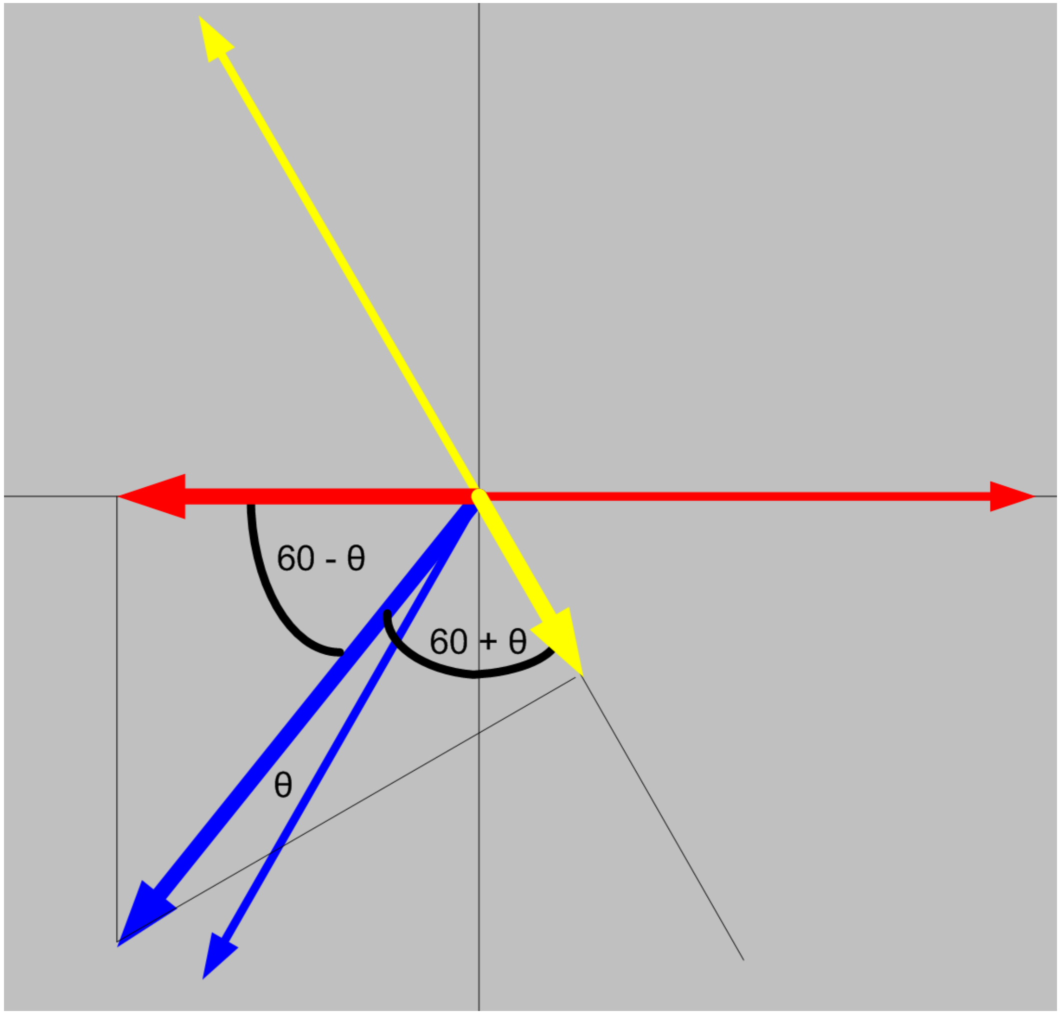
- In the previous example:
  - What are the “ASSUMPTIONS”?
  - When do we get errors?
- What would the “Right Answer” be?

$$P_{sys} = V_a I_a \cos(\theta_a) + V_b I_b \cos(\theta_b) + V_c I_c \cos(\theta_c)$$

- What did we measure?

$$P_{sys} = V_a [I_a \cos(\theta_a) - I_b \cos(\theta_b)] + V_c [I_c \cos(\theta_c) - I_b \cos(\theta_b)]$$

Blondel's Theorem



Blondel's Theorem

- Phase B power would be:
  - $P = V_b I_b \cos\theta$
- But we aren't measuring Vb
- What we are measuring is:
  - $I_b V_a \cos(60 - \theta) + I_b V_c \cos(60 + \theta)$
- $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$
- $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$
- So

Blondel's Theorem

- $Pb = Ib\ Va \cos(60 - \theta) + Ib\ Vc \cos(60 + \theta)$
- Applying the trig identity
  - $IbVa(\cos(60)\cos(\theta) + \sin(60)\sin(\theta))$
    $IbVc\ (\cos(60)\cos(\theta) - \sin(60)\sin(\theta))$
  - $Ib(Va+Vc)0.5\cos(\theta) + Ib(Vc-Va)\ 0.866\sin(\theta)$
- Assuming
  - Assume $Vb = Va = Vc$
  - And, they are exactly 120° apart
- $Pb = Ib(2Vb)(0.5\cos\theta) = IbVb\cos\theta$

Blondel's Theorem

- If Va ≠ Vb ≠ Vc then the error is
- %Error =

$$-Ib\{(Va+Vc)/(2Vb) - (Va-Vc)\ 0.866\sin(\theta)/(Vb\cos(\theta))$$

How big is this in reality? If

Va=117, Vb=120, Vc=119, PF=1 then E=-1.67%

Va=117, Vb=116, Vc=119, PF=.866 then E=-1.67%

Blondel’s Theorem

Power Measurements Handbook

Condition	% V	% I	Phase A				Phase B				non-Blondel
	Imb	Imb	V	ϕvan	I	ϕian	V	ϕvbn	I	ϕibn	% Err
All balanced	0	0	120	0	100	0	120	180	100	180	0.00%
Unbalanced voltages PF=1	18%	0%	108	0	100	0	132	180	100	180	0.00%
Unbalanced current PF=1	0%	18%	120	0	90	0	120	180	110	180	0.00%
Unbalanced V&I PF=1	5%	18%	117	0	90	0	123	180	110	180	-0.25%
Unbalanced V&I PF=1	8%	18%	110	0	90	0	120	180	110	180	-0.43%
Unbalanced V&I PF=1	8%	50%	110	0	50	0	120	180	100	180	-1.43%
Unbalanced V&I PF=1	18%	40%	108	0	75	0	132	180	125	180	-2.44%
Unbalanced voltages PF≠1 PFa = PFb	18%	0%	108	0	100	30	132	180	100	210	0.00%
Unbalanced current PF≠1 PFa = PFb	0%	18%	120	0	90	30	120	180	110	210	0.00%
Unbalanced V&I PF≠1 PFa = PFb	18%	18%	108	0	90	30	132	180	110	210	-0.99%
Unbalanced V&I PF≠1 PFa = PFb	18%	40%	108	0	75	30	132	180	125	210	-2.44%
Unbalanced voltages PF≠1 PFa ≠ PFb	18%	0%	108	0	100	60	132	180	100	210	-2.61%
Unbalanced current PF≠1 PFa ≠ PFb	0%	18%	120	0	90	60	120	180	110	210	0.00%
Unbalanced V&I PF≠1 PFa ≠ PFb	18%	18%	108	0	90	60	132	180	110	210	-3.46%
Unbalanced V&I PF≠1 PFa ≠ PFb	18%	40%	108	0	75	60	132	180	125	210	-4.63%

AC Theory – Power

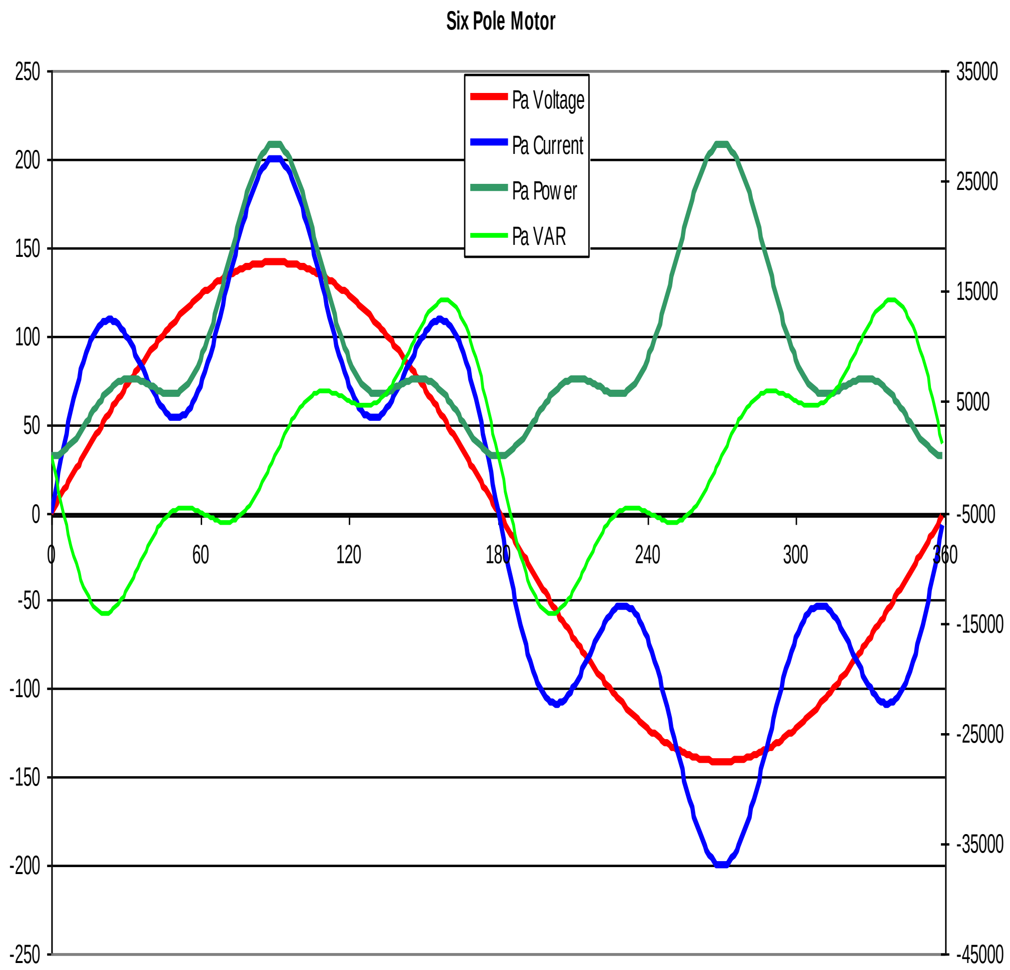
- Power is defined as $P = VI$
- Since the voltage and current at every point in time for an AC signal is different, we have to distinguish between instantaneous power and average power. Generally when we say “power” we mean average power.
- Average power is only defined over an integer number of cycles.

Harmonics
Curse of the Modern World

- Every thing discussed so far was based on “Linear” loads.
  - For linear loads the current is always a simple sine wave. Everything we have discussed is true.
- For nearly a century after AC power was in use ALL loads were linear.
- Today, many loads are NON-LINEAR.

Harmonic Load Waveform

Eq.#	Quantity	Phase A
1	V(rms) (Direct Sum)	100
2	I(rms) (Direct Sum)	108
3	V(rms) (Fourier)	100
4	I(rms) (Fourier)	108
5	Pa = (∫ V(t)I(t)dt)	10000
6	Pb = ½∑V*n*In**cos**(θ)	10000
7	Q = ½∑VnIn**sin**(θ)	0.000
8	Sa = Sqrt(P^2 +Q^2)	10000
9	Sb = Vrms*Irms(DS)	10833
10	Sc = Vrms*Irms(F)	10833
13	PF = Pa/Sa	1.000
14	PF = Pb/Sb	0.923
15	PF = Pb/Sc	0.923



V = 100Sin(ωt)   I = 100Sin(ωt) + 42Sin(5 ωt)

Harmonic Load Waveform

Eq.#	Quantity	Phase A
1	V(rms) (Direct Sum)	100
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8	Sa = Sqrt(P^2 +Q^2)	10000
9	Sb = Vrms*Irms(DS)	10833
10	Sc = Vrms*Irms(F)	10833
13	PF = Pa/Sa	1.000
14	PF = Pb/Sb	0.923
15	PF = Pb/Sc	0.923

- Important things to note:
  - Because the voltage is NOT distorted, the harmonic in the current does not contribute to active power.
  - It does contribute to the Apparent power.
  - Does the Power Triangle hold

$$S ? = \sqrt{P^2 + Q^2}$$

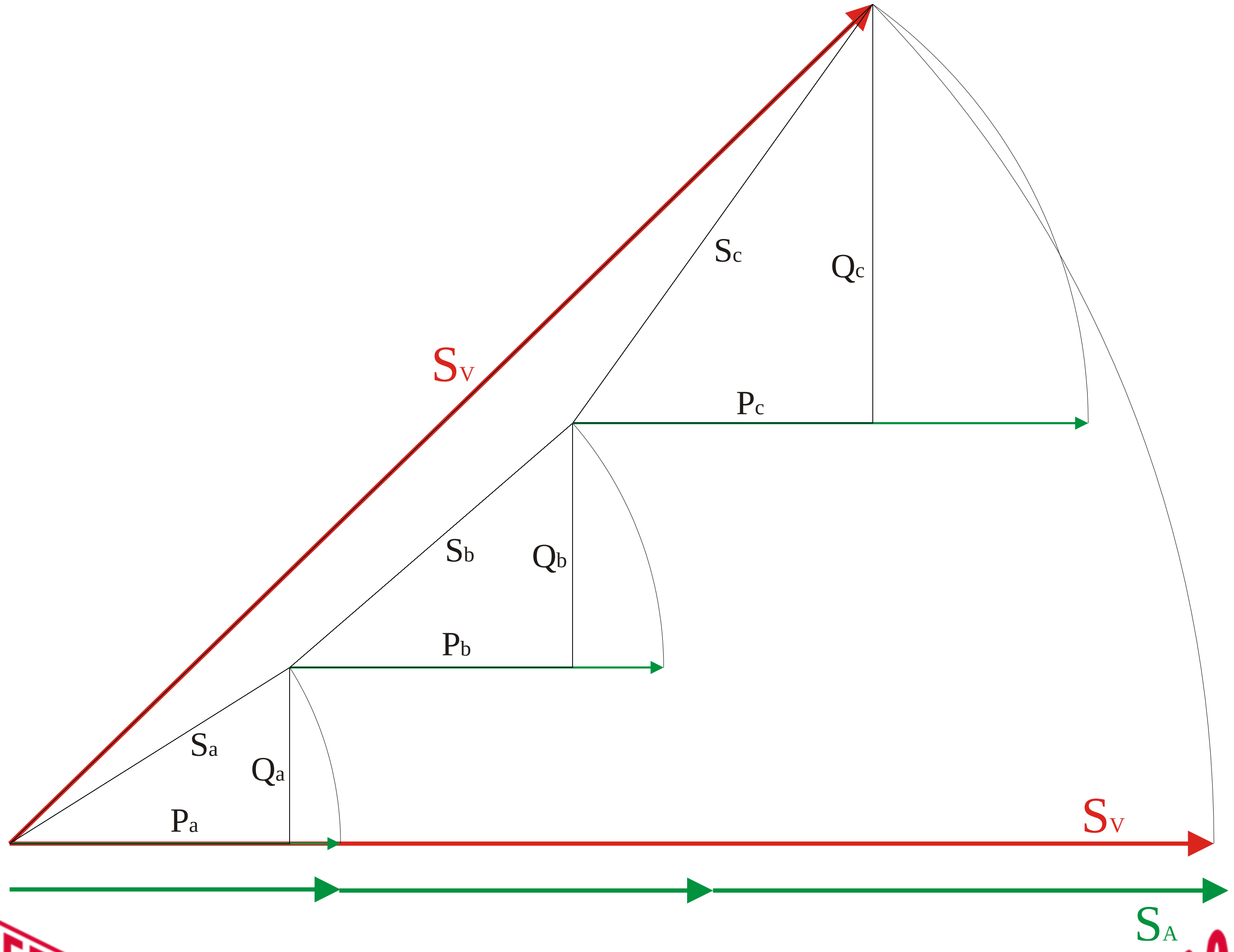
  - There is considerable disagreement about the definition of various power quantities when harmonics are present.

V = 100Sin(ωt)   I = 100Sin(ωt) + 42Sin(5 ωt)

3 Phase Power Measurement

- We have discussed how to measure and view power quantities (W, VARs, VA) in a single phase case.
- How do we combine them in a multi-phase system?
- Two common approaches:
  - Arithmetic
  - Vectorial

3 Phase Power Measurement

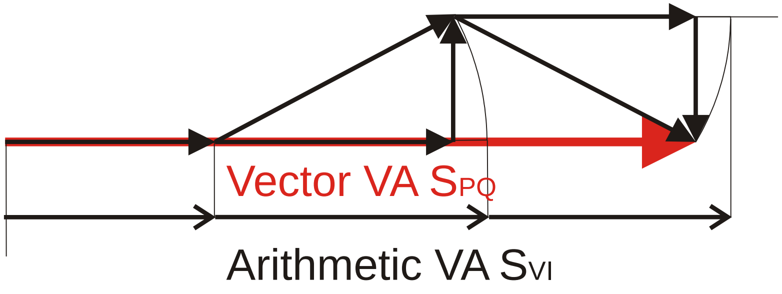


3 Phase Power Measurement

- VAR and VA calculations can lead to some strange results:
  - If we define

$$VA = \sqrt{(W_A + W_B + W_C)^2 + (Q_A + Q_B + Q_C)^2}$$

PH	W	Q	VA
A	100	0	100
B	120	55	132
C	120	-55	132
Arithmetic VA			364
Vector VA			340



Questions and Discussion

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